CFVVI – Prova 2 – Turma 2 15/10/2025

Gabarito

1. Não é permitido o uso de celulares, calculadoras ou dispositivos eletrônicos!
2. A avaliação é individual e não é permitida consulta!
3. Respeite as margens do papel e não utilize caneta vermelha ou corretivo!
4. Todas as respostas devem ser devidamente justificadas!
5. Não pule passagens e use a notação matemática correta!
6. O resultado final correto não significa nada se o procedimento estiver errado!

1 [30%] Use a definição para calcular a derivada parcial de $f(x, y) = x^2y + 3xy^2$ por x

Pela definição,

$$\frac{\partial f}{\partial x}(x, y) = \lim_{h\to 0} \frac{f(x + h, y) - f(x, y)}{h}$$

Avaliando $f(x + h, y)$

$$\begin{aligned} f(x, y) &= x^2y + 3xy^2 \\ f(x + h, y) &= (x + h)^2y + 3(x + h)y^2 \\ &= (x^2 + 2xh + h^2)y + 3xy^2 + 3y^2h \\ &= x^2y + 2xyh + yh^2 + 3xy^2 + 3y^2h \end{aligned}$$

Portanto,

$$\begin{aligned} f(x + h, y) - f(x,y) &= (x^2y + 2xyh + yh^2 + 3xy^2 + 3y^2h) - (x^2y + 3xy^2) \\ &= 2xyh + yh^2 + 3y^2h \\ &= (2xy + 3y^2 + yh)h \end{aligned}$$

Dividindo por h

$$\frac{f(x + h, y) - f(x, y)}{h} = 2xy + 3y^2 + hy$$

Tomando o limite

$$\begin{aligned} \frac{\partial f}{\partial x}(x,y) &= \lim_{h\to 0} \frac{f(x + h,y) - f(x,y)}{h} \\ &= \lim_{h\to 0} (2xy + 3y^2 + hy) \\ &= 2xy + 3y^2 \end{aligned}$$

2 [40%] Calcule $\dfrac{\partial^2 f}{\partial x \partial y}(\pi, 1)$, respeitando a ordem descrita na notação, de $f(x,y) = x^2 \cos\left(xy^2\right)$

$$\begin{aligned}
\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y}\left[x^2 \cos\left(xy^2\right)\right] \\
&= x^2 \, \frac{\partial}{\partial y} \cos\left(xy^2\right) \\
&= x^2 \left[-\operatorname{sen}\left(xy^2\right)\right] \frac{\partial}{\partial y}\left(xy^2\right) \\
&= -x^2 \operatorname{sen}\left(xy^2\right) \; x\frac{\partial y^2}{\partial y} \\
&= -x^3 \operatorname{sen}\left(xy^2\right) \; 2y \\
&= -2yx^3 \operatorname{sen}\left(xy^2\right)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 f}{\partial x \partial y} &= \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) \\
&= \frac{\partial}{\partial x}\left(-2yx^3 \operatorname{sen}\left(xy^2\right)\right) \\
&= -2y\frac{\partial}{\partial x}\left(x^3 \operatorname{sen}\left(xy^2\right)\right) \\
&= -2y\left[\frac{\partial x^3}{\partial x} \operatorname{sen}\left(xy^2\right) + x^3 \frac{\partial}{\partial x} \operatorname{sen}\left(xy^2\right)\right] \\
&= -2y\left[3x^2 \operatorname{sen}\left(xy^2\right) + x^3 \cos\left(xy^2\right) \frac{\partial}{\partial x}\left(xy^2\right)\right] \\
&= -2y\left[3x^2 \operatorname{sen}\left(xy^2\right) + x^3 \cos\left(xy^2\right) y^2\right] \\
&= -2yx^2\left[3 \operatorname{sen}\left(xy^2\right) + xy^2 \cos\left(xy^2\right)\right]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 f}{\partial x \partial y}(\pi, 1) &= -2yx^2\left[3 \operatorname{sen}\left(xy^2\right) + xy^2 \cos\left(xy^2\right)\right]\Big|_{(\pi,1)} \\
&= -2 \times 1 \times \pi^2 \left[3 \operatorname{sen}\left(\pi 1^2\right) + \pi 1 \cos\left(\pi 1^2\right)\right] \\
&= -2\pi^2 \left[3 \operatorname{sen}(\pi) + \pi \cos(\pi)\right] \\
&= -2\pi^2 \left[0 - \pi\right] \\
&= 2\pi^3
\end{aligned}$$

3 [30%] Utilize a derivação implícita para encontrar $\dfrac{\partial z}{\partial y}$, assumindo que a equação $xe^{yz}+y^2z=1$ define z como função de x e y

Utilizando a fórmula com $F(x,y,z) = xe^{yz} + y^2z - 1$

$$\begin{aligned}
F_y(x,y,z) &= \frac{\partial F}{\partial y}(x,y,z) \\
&= \frac{\partial}{\partial y}\left(xe^{yz} + y^2z - 1\right) \\
&= \frac{\partial}{\partial y}\left(xe^{yz}\right) + \frac{\partial}{\partial y}\left(y^2z\right) - \frac{\partial 1}{\partial y} \\
&= x\frac{\partial}{\partial y}\left(e^{yz}\right) + z\frac{\partial}{\partial y}\left(y^2\right) \\
&= xe^{yz}\frac{\partial}{\partial y}\left(yz\right) + z2y \\
&= xze^{yz} + 2yz \\
F_z(x,y,z) &= \frac{\partial F}{\partial z}(x,y,z) \\
&= \frac{\partial}{\partial z}\left(xe^{yz} + y^2z - 1\right) \\
&= \frac{\partial}{\partial z}\left(xe^{yz}\right) + \frac{\partial}{\partial z}\left(y^2z\right) - \frac{\partial 1}{\partial z} \\
&= x\frac{\partial e^{yz}}{\partial z} + y^2\frac{\partial z}{\partial z} \\
&= xe^{yz}\frac{\partial yz}{\partial z} + y^2 \\
&= xye^{yz} + y^2
\end{aligned}$$

Portanto,

$$\frac{\partial z}{\partial y}(x,y,z) = -\frac{F_y(x,y,z)}{F_z(x,y,z)} = -\frac{xze^{yz} + 2yz}{xye^{yz} + y^2}$$