

GABARITO

1. Não é permitido o uso de celulares, calculadoras ou dispositivos eletrônicos!
2. A avaliação é individual e não é permitida consulta!
3. Respeite as margens do papel e não utilize caneta vermelha ou corretivo!
4. Todas as respostas devem ser devidamente justificadas!
5. Não pule passagens, escreva ao menos uma, e use a notação matemática correta!
6. O resultado final correto não significa nada se o procedimento estiver errado!

1 [50%] Uma trajetória elíptica é descrita pelas equações paramétricas

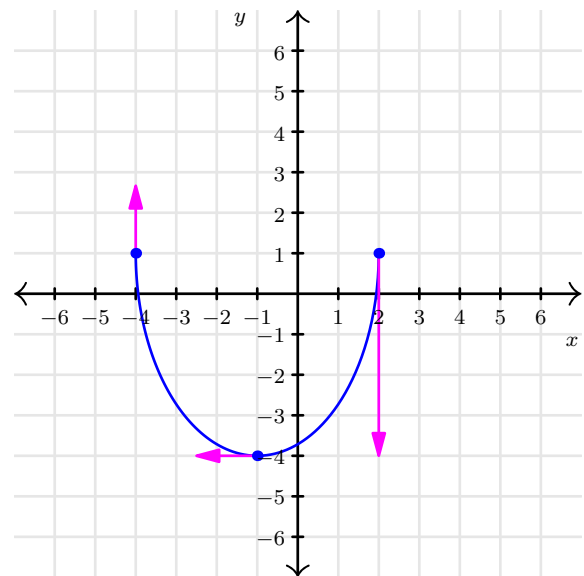
$$x(t) = 3 \operatorname{sen}(\sqrt{t}) - 1$$

$$y(t) = 5 \cos(\sqrt{t}) + 1$$

$$0 \leq t \leq \pi^2$$

- [10%] determine os pontos da curva com $t_1 = \frac{\pi^2}{4}$, $t_2 = \pi^2$ e $t_3 = \frac{9\pi^2}{4}$
- [15%] calcule a expressão para o vetor tangente à curva
- [15%] avalie o vetor tangente à curva nos pontos t_1 , t_2 e t_3
- [10%] esboce a curva e os vetores tangentes calculados

Para esboçar dos vetores, multiplique-os por π



a) Calculando os pontos

$$\begin{aligned} x\left(\frac{\pi^2}{4}\right) &= \left(3 \operatorname{sen}(\sqrt{t}) - 1\right) \Big|_{\frac{\pi^2}{4}} \\ &= 3 \operatorname{sen}\left(\sqrt{\frac{\pi^2}{4}}\right) - 1 \\ &= 3 \operatorname{sen}\left(\frac{\pi}{2}\right) - 1 \\ &= 3 \times 1 - 1 \\ &= 2 \end{aligned}$$

$$\begin{aligned} y\left(\frac{\pi^2}{4}\right) &= \left(5 \cos(\sqrt{t}) + 1\right) \Big|_{\frac{\pi^2}{4}} \\ &= 5 \cos\left(\sqrt{\frac{\pi^2}{4}}\right) + 1 \\ &= 5 \cos\left(\frac{\pi}{2}\right) + 1 \\ &= 5 \times 0 + 1 \\ &= 1 \end{aligned}$$

$$\begin{aligned}
x\left(\pi^2\right) &= \left(3 \operatorname{sen}\left(\sqrt{t}\right)-1\right)\Big|_{\pi^2} \\
&= 3 \operatorname{sen}\left(\sqrt{\pi^2}\right)-1 \\
&= 3 \operatorname{sen}(\pi)-1 \\
&= 3 \times 0-1 \\
&= -1
\end{aligned}$$

$$\begin{aligned}
y\left(\pi^2\right) &= \left(5 \cos \left(\sqrt{t}\right)+1\right)\Big|_{\pi^2} \\
&= 5 \cos \left(\sqrt{\pi^2}\right)+1 \\
&= 5 \cos (\pi)+1 \\
&= 5(-1)+1 \\
&= -4
\end{aligned}$$

$$\begin{aligned}
x\left(\frac{9 \pi^2}{4}\right) &= \left(3 \operatorname{sen}\left(\sqrt{t}\right)-1\right)\Big|_{\frac{9 \pi^2}{4}} \\
&= 3 \operatorname{sen}\left(\sqrt{\frac{9 \pi^2}{4}}\right)-1 \\
&= 3 \operatorname{sen}\left(\frac{3 \pi}{2}\right)-1 \\
&= 3(-1)-1 \\
&= -4
\end{aligned}$$

$$\begin{aligned}
y\left(\frac{9 \pi^2}{4}\right) &= \left(5 \cos \left(\sqrt{t}\right)+1\right)\Big|_{\frac{9 \pi^2}{4}} \\
&= 5 \cos \left(\sqrt{\frac{9 \pi^2}{4}}\right)+1 \\
&= 5 \cos \left(\frac{3 \pi}{2}\right)+1 \\
&= 5 \times 0+1 \\
&= 1
\end{aligned}$$

b) O vetor tangente é $v(t)=\left(\begin{array}{c} \frac{dx}{dt} \\ \frac{dy}{dt} \end{array}\right)$

$$\begin{aligned}
\frac{dx}{dt} &= \frac{d}{dt}\left[3 \operatorname{sen}\left(\sqrt{t}\right)-1\right] \\
&= 3 \frac{d}{dt}\left[\operatorname{sen}\left(\sqrt{t}\right)\right] \\
&= 3 \cos \left(\sqrt{t}\right) \frac{d}{dt}\left[\sqrt{t}\right] \\
&= 3 \cos \left(\sqrt{t}\right) \frac{1}{2 \sqrt{t}} \\
&= \frac{3 \cos \left(\sqrt{t}\right)}{2 \sqrt{t}}
\end{aligned}$$

$$\begin{aligned}
\frac{dy}{dt} &= \frac{d}{dt}\left[5 \cos \left(\sqrt{t}\right)+1\right] \\
&= 5 \frac{d}{dt}\left[\cos \left(\sqrt{t}\right)\right] \\
&= -5 \operatorname{sen}\left(\sqrt{t}\right) \frac{d}{dt}\left[\sqrt{t}\right] \\
&= -5 \operatorname{sen}\left(\sqrt{t}\right) \frac{1}{2 \sqrt{t}} \\
&= \frac{-5 \operatorname{sen}\left(\sqrt{t}\right)}{2 \sqrt{t}}
\end{aligned}$$

$$v(t)=\left(\begin{array}{c} \frac{3 \cos \left(\sqrt{t}\right)}{2 \sqrt{t}} \\ \frac{-5 \operatorname{sen}\left(\sqrt{t}\right)}{2 \sqrt{t}} \end{array}\right)=\frac{1}{2 \sqrt{t}}\left(\begin{array}{c} 3 \cos \left(\sqrt{t}\right) \\ -5 \operatorname{sen}\left(\sqrt{t}\right) \end{array}\right)$$

c) Avaliando o vetor tangente nos pontos $t_1 = \frac{\pi^2}{4}$, $t_2 = \pi^2$ e $t_3 = \frac{9\pi^2}{4}$

$$\begin{aligned} v\left(\frac{\pi^2}{4}\right) &= \frac{1}{2\sqrt{t}} \begin{pmatrix} 3 \cos(\sqrt{t}) \\ -5 \operatorname{sen}(\sqrt{t}) \end{pmatrix} \Big|_{\frac{\pi^2}{4}} = \frac{1}{2\sqrt{\frac{\pi^2}{4}}} \begin{pmatrix} 3 \cos\left(\sqrt{\frac{\pi^2}{4}}\right) \\ -5 \operatorname{sen}\left(\sqrt{\frac{\pi^2}{4}}\right) \end{pmatrix} \\ &= \frac{1}{2\frac{\pi}{2}} \begin{pmatrix} 3 \cos\left(\frac{\pi}{2}\right) \\ -5 \operatorname{sen}\left(\frac{\pi}{2}\right) \end{pmatrix} = \frac{1}{\pi} \begin{pmatrix} 3 \cos\left(\frac{\pi}{2}\right) \\ -5 \operatorname{sen}\left(\frac{\pi}{2}\right) \end{pmatrix} = \frac{1}{\pi} \begin{pmatrix} 3 \times 0 \\ -5 \times 1 \end{pmatrix} = \frac{1}{\pi} \begin{pmatrix} 0 \\ -5 \end{pmatrix} \end{aligned}$$

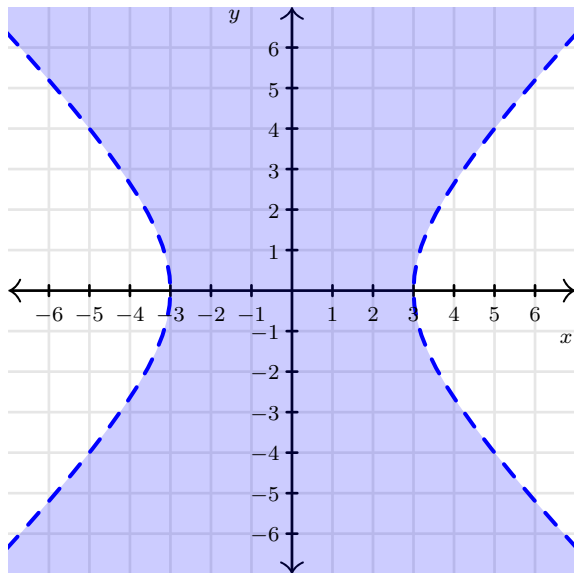
$$\begin{aligned} v\left(\pi^2\right) &= \frac{1}{2\sqrt{t}} \begin{pmatrix} 3 \cos(\sqrt{t}) \\ -5 \operatorname{sen}(\sqrt{t}) \end{pmatrix} \Big|_{\pi^2} = \frac{1}{2\sqrt{\pi^2}} \begin{pmatrix} 3 \cos(\sqrt{\pi^2}) \\ -5 \operatorname{sen}(\sqrt{\pi^2}) \end{pmatrix} \\ &= \frac{1}{2\pi} \begin{pmatrix} 3 \cos(\pi) \\ -5 \operatorname{sen}(\pi) \end{pmatrix} = \frac{1}{2\pi} \begin{pmatrix} 3(-1) \\ -5 \times 0 \end{pmatrix} = \frac{1}{2\pi} \begin{pmatrix} -3 \\ 0 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} v\left(\frac{9\pi^2}{4}\right) &= \frac{1}{2\sqrt{t}} \begin{pmatrix} 3 \cos(\sqrt{t}) \\ -5 \operatorname{sen}(\sqrt{t}) \end{pmatrix} \Big|_{\frac{9\pi^2}{4}} = \frac{1}{2\sqrt{\frac{9\pi^2}{4}}} \begin{pmatrix} 3 \cos\left(\sqrt{\frac{9\pi^2}{4}}\right) \\ -5 \operatorname{sen}\left(\sqrt{\frac{9\pi^2}{4}}\right) \end{pmatrix} \\ &= \frac{1}{2\frac{3\pi}{2}} \begin{pmatrix} 3 \cos\left(\frac{3\pi}{2}\right) \\ -5 \operatorname{sen}\left(\frac{3\pi}{2}\right) \end{pmatrix} = \frac{1}{3\pi} \begin{pmatrix} 3 \cos\left(\frac{3\pi}{2}\right) \\ -5 \operatorname{sen}\left(\frac{3\pi}{2}\right) \end{pmatrix} = \frac{1}{3\pi} \begin{pmatrix} 3 \times 0 \\ -5(-1) \end{pmatrix} = \frac{1}{3\pi} \begin{pmatrix} 0 \\ 5 \end{pmatrix} \end{aligned}$$

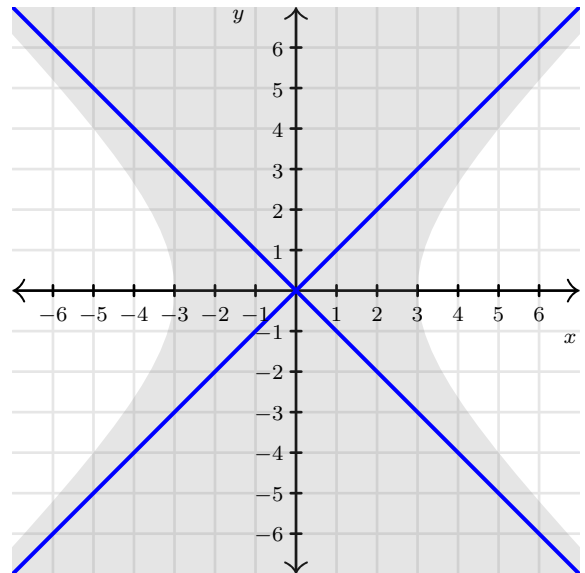
2 [50%] Considerando a função $f(x, y) = \sqrt{\ln(10 - x^2 + y^2)}$

- (a) [15%] determine o domínio de f
- (b) [15%] esboce o domínio
- (c) [10%] determine a curva de nível que passa pelo ponto $(0, 0)$
- (d) [10%] esboce a curva de nível que passa pelo ponto $(0, 0)$

Domínio



Curva de nível



a) Para calcular a raiz precisamos que

$$\ln(10 - x^2 + y^2) \geq 0$$

$$10 - x^2 + y^2 \geq 1$$

$$-x^2 + y^2 \geq 9$$

$$x^2 - y^2 \leq 9$$

$$\frac{x^2}{3^2} - \frac{y^2}{3^2} \leq 1$$

ou seja, os pontos “entre os lados” da hipérbole.
Assim, o domínio é

$$D_f = \{(x, y) \in \mathbb{R}^2 \mid x^2 - y^2 < 9\}$$

c) A curva de nível que passa pelo ponto $(0, 0)$
tem o valor

$$c = f(0, 0)$$

$$= \sqrt{\ln(10 - x^2 + y^2)} \Big|_{(0,0)}$$

$$= \sqrt{\ln(10 - 0^2 + 0^2)}$$

$$= \sqrt{\ln 10}$$

A curva de nível associada é

$$f(x, y) = c$$

$$\sqrt{\ln(10 - x^2 + y^2)} = \sqrt{\ln 10}$$

$$\ln(10 - x^2 + y^2) = \ln 10$$

$$10 - x^2 + y^2 = 10$$

$$-x^2 + y^2 = 0$$

$$y^2 = x^2$$

$$y = \pm x$$

A curva de nível corresponde as retas

$$y = x \quad \text{e} \quad y = -x$$