Vetor Tangente

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Cálculo de Funções de Várias Variáveis – I

https://material-didatico.github.io/pages/cfvv1

11 de março de 2026

Conteúdo

Curva Paramétrica

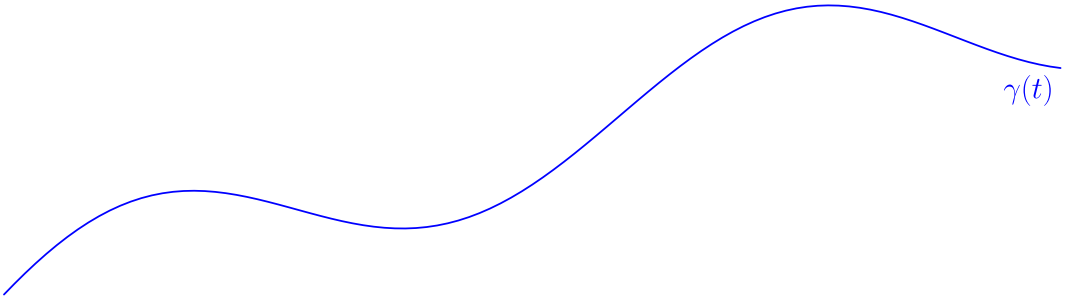
x e y são dados como funções

$$x = f(t) \qquad y = g(t) \qquad t \in I$$

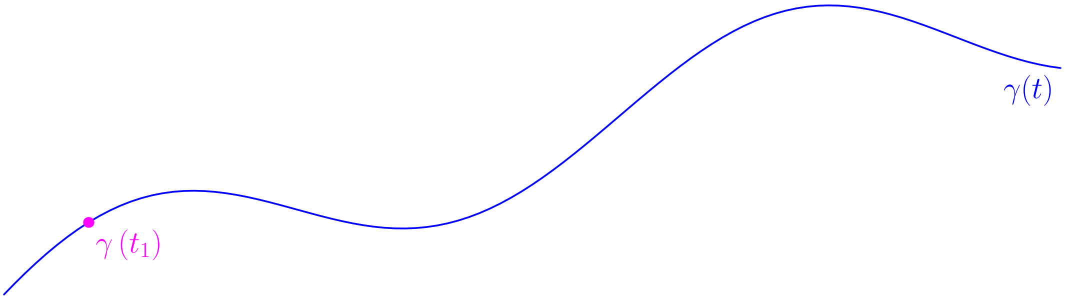
Vetorialmente

$$\gamma(t) = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} f(t) \\ g(t) \end{pmatrix}$$

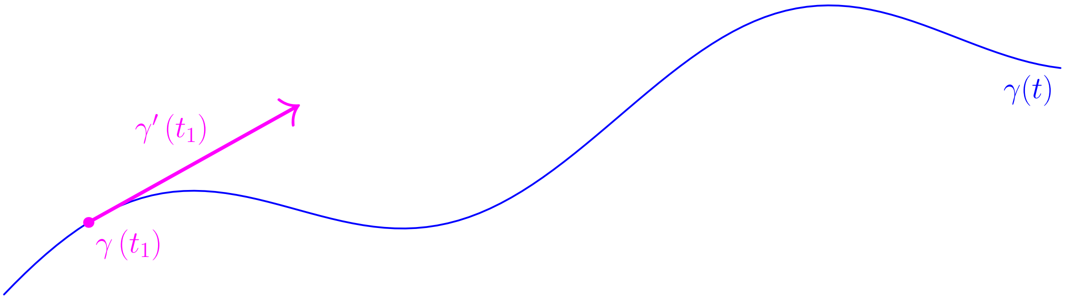
Vetor Tangente



Vetor Tangente



Vetor Tangente



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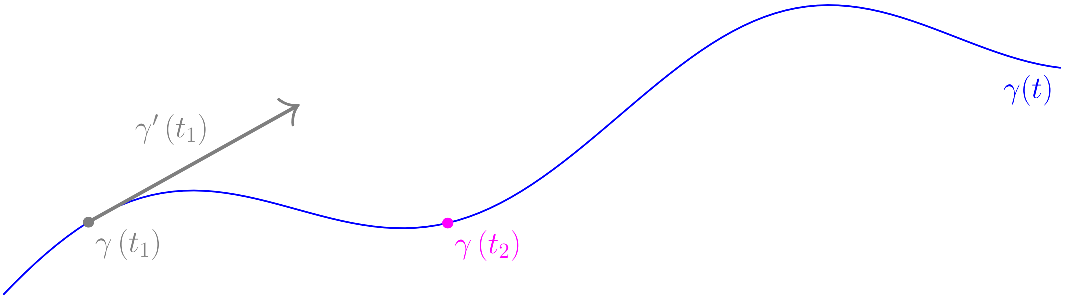

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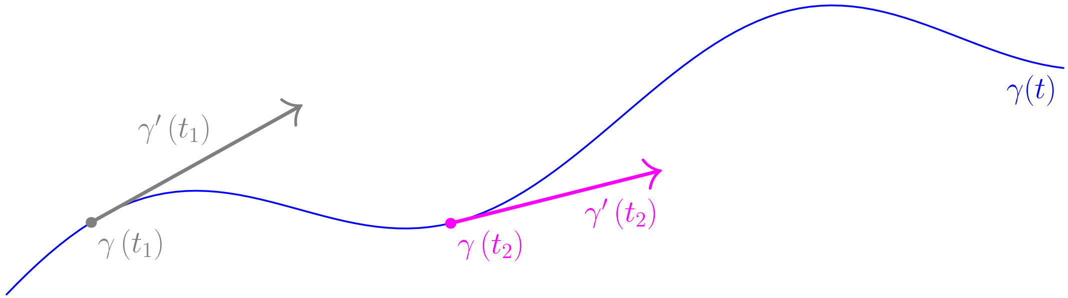

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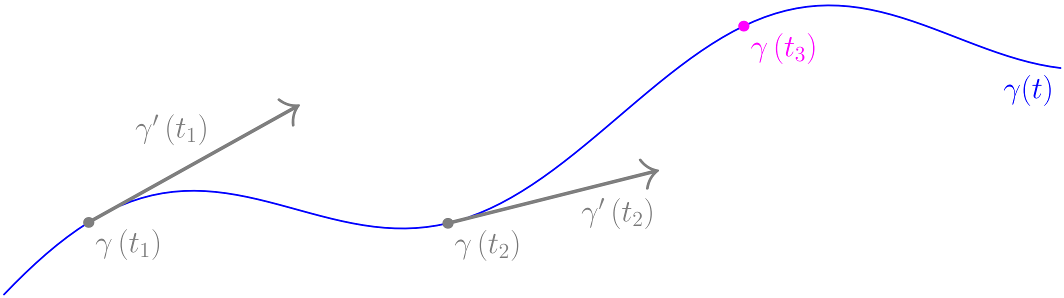
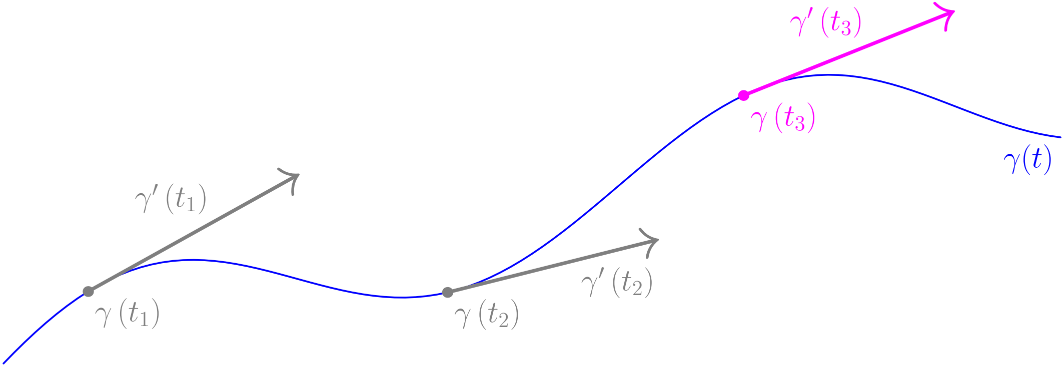


Vetor Tangente



Vetor Tangente

Se

$$\gamma(t) = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} f(t) \\ g(t) \end{pmatrix} \qquad t \in I$$

e f e g forem deriváveis em t, o vetor tangente é

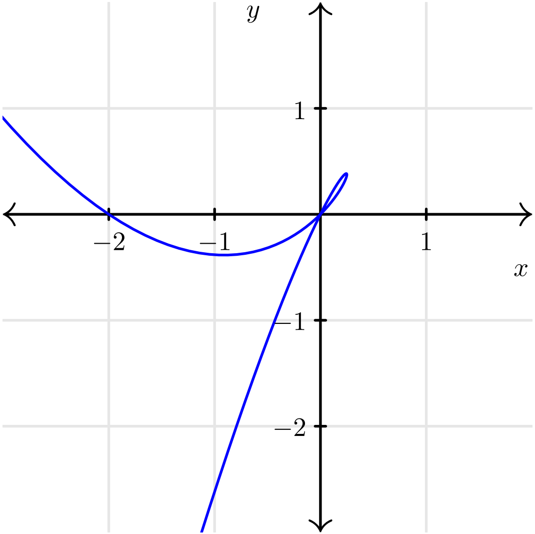
$$\gamma'(t) = \frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} \dfrac{df}{dt} \\[2ex] \dfrac{dg}{dt} \end{pmatrix} = \begin{pmatrix} f'(t) \\ g'(t) \end{pmatrix}$$

Conteúdo

Exemplo 1

Calcule o vetor tangente à curva

$$x = t - t^2 \qquad y = t - t^3$$



Exemplo 1 – Solução

Calculando $\dfrac{dx}{dt}$ e $\dfrac{dy}{dt}$

$$\frac{dx}{dt} = \frac{d}{dt}\left(t - t^2\right) = 1 - 2t$$

$$\frac{dy}{dt} = \frac{d}{dt}\left(t - t^3\right) = 1 - 3t^2$$

Vetor tangente

$$v(t) = \begin{pmatrix} \dfrac{dx}{dt} \\[2ex] \dfrac{dy}{dt} \end{pmatrix} = \begin{pmatrix} 1 - 2t \\ 1 - 3t^2 \end{pmatrix}$$

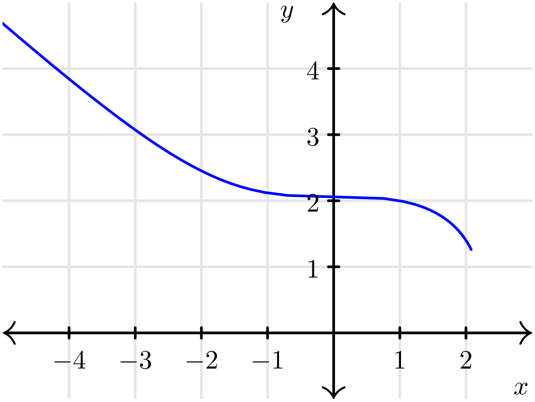
Exemplo 2

Use derivação implícita para calcular o vetor tangente da curva paramétrica

$$\gamma(t) = \big(x(t), y(t)\big)$$

definida pelas equações

$$x^3 + 2t^2 = 9 \qquad 2y^3 - 3t^2 = 4$$



Exemplo 2 – Derivação Explicita ou Implícita

Nesse caso podemos isolar x e y

$$x = \sqrt[3]{9 - 2t^2} \qquad y = \sqrt[3]{\frac{4 + 3t^2}{2}}$$

Como resolver se não fosse possível?

Exemplo 2 – Solução

$$
\begin{aligned}
x^3 + 2t^2 &= 9 \\
x^3 &= 9 - 2t^2 \\
\frac{d}{dt}\left(x^3\right) &= \frac{d}{dt}\left(9 - 2t^2\right) \\
3x^2 \frac{dx}{dt} &= 0 - 2 \times 2t \\
\frac{dx}{dt} &= \frac{-4t}{3x^2}
\end{aligned}
$$

$$
\begin{aligned}
2y^3 - 3t^2 &= 4 \\
2y^3 &= 4 + 3t^2 \\
\frac{d}{dt}\left(2y^3\right) &= \frac{d}{dt}\left(4 + 3t^2\right) \\
2 \times 3y^2 \frac{dy}{dt} &= 0 + 3 \times 2t \\
\frac{dy}{dt} &= \frac{6t}{6y^2} = \frac{t}{y^2}
\end{aligned}
$$

Exemplo 2 – Solução

Vetor tangente

$$\gamma'(t) = \begin{pmatrix} \dfrac{-4t}{3x^2} \\ \\ \dfrac{t}{y^2} \end{pmatrix}$$

Exemplo 3

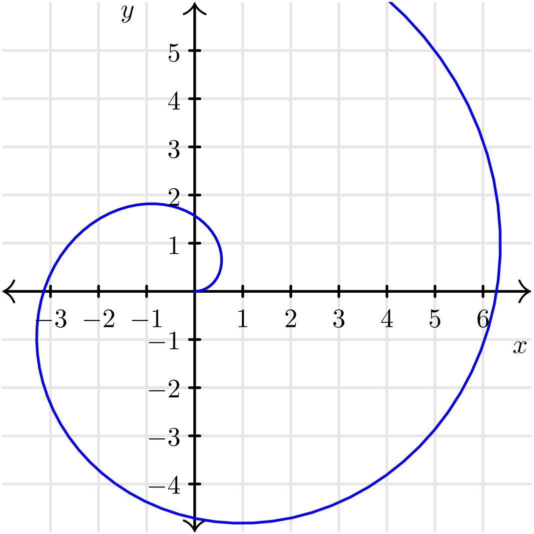
Calcule o vetor tangente à curva

$$x = t\cos(t)$$

$$y = t\,\mathrm{sen}(t)$$

$$0 \le t < \infty$$

quando $t = \dfrac{\pi}{2}$



Exemplo 3 – Solução

Derivada da coordenada x

$$
\begin{aligned}
x'(t) &= \frac{dx}{dt} \\
&= \frac{d}{dt} t\cos(t) \\
&= 1 \times \cos(t) + t(-\operatorname{sen}(t)) \\
&= \cos(t) - t\operatorname{sen}(t)
\end{aligned}
$$

Derivada da coordenada y

$$
\begin{aligned}
y'(t) &= \frac{dy}{dt} \\
&= \frac{d}{dt} t\operatorname{sen}(t) \\
&= 1 \times \operatorname{sen}(t) + t\cos(t) \\
&= \operatorname{sen}(t) + t\cos(t)
\end{aligned}
$$

Exemplo 3 – Solução

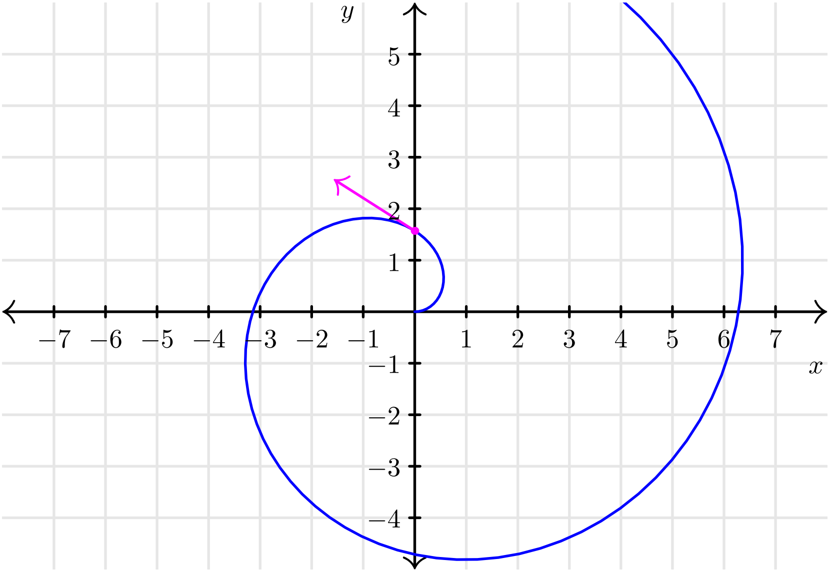
Avaliando as funções derivada em $t = \dfrac{\pi}{2}$

$$x'\left(\frac{\pi}{2}\right) = (\cos(t) - t\,\mathrm{sen}(t))\Big|_{\frac{\pi}{2}} = \cos\left(\frac{\pi}{2}\right) - \frac{\pi}{2}\,\mathrm{sen}\left(\frac{\pi}{2}\right) = 0 - \frac{\pi}{2} \times 1 = -\frac{\pi}{2}$$

$$y'\left(\frac{\pi}{2}\right) = (\mathrm{sen}(t) + t\cos(t))\Big|_{\frac{\pi}{2}} = \mathrm{sen}\left(\frac{\pi}{2}\right) + \frac{\pi}{2}\cos\left(\frac{\pi}{2}\right) = 1 + \frac{\pi}{2} \times 0 = 1$$

O vetor tangente é $\begin{bmatrix} -\,{}^{\pi}\!/_{2} \\ 1 \end{bmatrix}$

Exemplo 3 – Solução

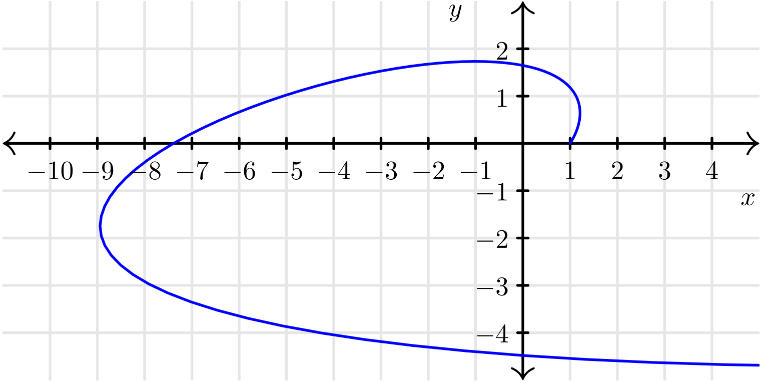


Exemplo 4

Calcule o vetor tangente à curva

$$\begin{cases} x = e^{2t}\cos(\pi t) \\ y = e^{t}\,\mathrm{sen}(\pi t) \end{cases}$$

em $t = \dfrac{3}{4}$



Exemplo 4 – Solução

Derivada da coordenada x

$$
\begin{aligned}
x'(t) &= \frac{dx}{dt} \\
&= \frac{d}{dt}\left(e^{2t}\cos(\pi t)\right) \\
&= \frac{d}{dt}\left(e^{2t}\right)\cos(\pi t) + e^{2t}\frac{d}{dt}\left(\cos(\pi t)\right) \\
&= e^{2t}\frac{d}{dt}(2t)\cos(\pi t) - e^{2t}\operatorname{sen}(\pi t)\frac{d}{dt}(\pi t) \\
&= 2e^{2t}\cos(\pi t) - \pi e^{2t}\operatorname{sen}(\pi t)
\end{aligned}
$$

Exemplo 4 – Solução

Derivada da coordenada y

$$\begin{aligned}
y'(t) &= \frac{dy}{dt} \\
&= \frac{d}{dt}\left(e^t \operatorname{sen}(\pi t)\right) \\
&= \frac{d}{dt}\left(e^t\right)\operatorname{sen}(\pi t) + e^t \frac{d}{dt}\left(\operatorname{sen}(\pi t)\right) \\
&= e^t \operatorname{sen}(\pi t) + e^t \cos(\pi t)\frac{d}{dt}(\pi t) \\
&= e^t \operatorname{sen}(\pi t) + \pi e^t \cos(\pi t)
\end{aligned}$$

Exemplo 4 – Solução

Avaliando a derivada de $x(t)$ em $t = {}^3\!/_4$

$$
\begin{aligned}
x'\left(\frac{3}{4}\right) &= \left[2e^{2t}\cos(\pi t) - \pi e^{2t}\,\mathrm{sen}(\pi t)\right]\Big|_{t={}^3\!/_4} \\
&= 2e^{2\frac{3}{4}}\cos\left(\pi\frac{3}{4}\right) - \pi e^{2\frac{3}{4}}\,\mathrm{sen}\left(\pi\frac{3}{4}\right) \\
&= 2e^{{}^3\!/_2}\cos\left(\frac{3\pi}{4}\right) - \pi e^{{}^3\!/_2}\,\mathrm{sen}\left(\frac{3\pi}{4}\right) \\
&= 2e^{{}^3\!/_2}\left(-\frac{\sqrt{2}}{2}\right) - \pi e^{{}^3\!/_2}\frac{\sqrt{2}}{2} \\
&= -\sqrt{2}\left(1 + \frac{\pi}{2}\right)e^{{}^3\!/_2}
\end{aligned}
$$

Exemplo 4 – Solução

Avaliando a derivada de $y(t)$ em $t = {}^{3}/_{4}$

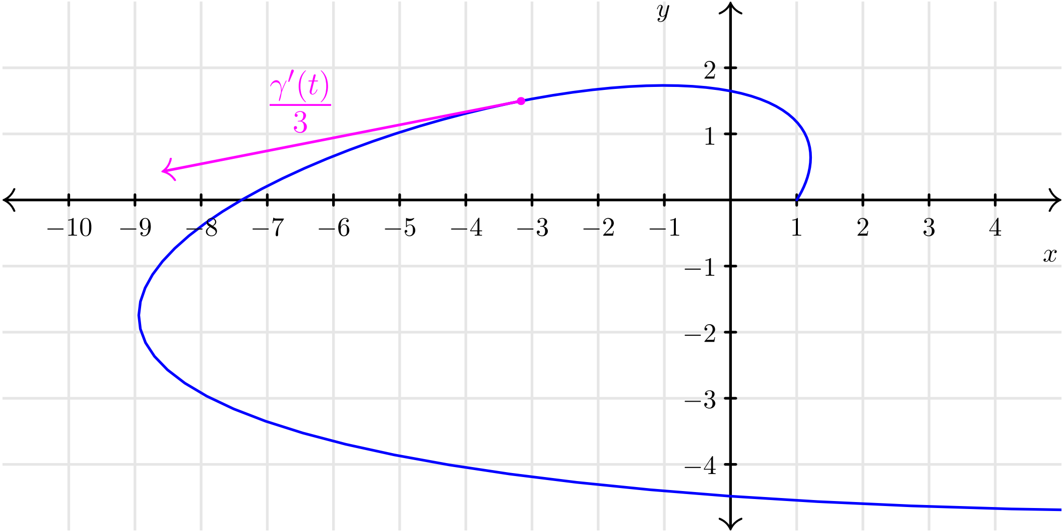
$$
\begin{aligned}
y'\left(\frac{3}{4}\right) &= \left(e^{t}\operatorname{sen}(\pi t) + \pi e^{t}\cos(\pi t)\right)\Big|_{t={}^{3}/_{4}} \\
&= e^{{}^{3}/_{4}}\operatorname{sen}\left(\frac{3\pi}{4}\right) + \pi e^{{}^{3}/_{4}}\cos\left(\frac{3\pi}{4}\right) \\
&= \frac{e^{{}^{3}/_{4}}}{\sqrt{2}} - \pi\frac{e^{{}^{3}/_{4}}}{\sqrt{2}} \\
&= \frac{1-\pi}{\sqrt{2}}e^{{}^{3}/_{4}}
\end{aligned}
$$

Exemplo 4 – Solução

O vetor tangente no ponto $t = {}^3\!/_4$ é

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -\sqrt{2}\left(1 + \dfrac{\pi}{2}\right) e^{3/2} \\ \\ \dfrac{1-\pi}{\sqrt{2}} e^{3/4} \end{pmatrix}$$

Exemplo 4 – Solução



Exemplo 5

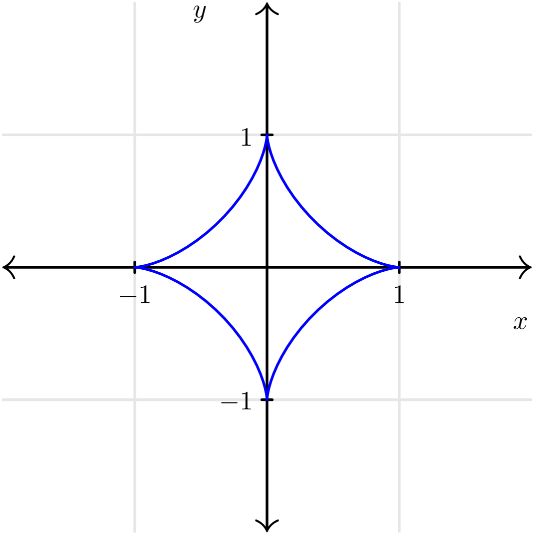
Encontre o vetor tangente ao astroide

$$x = \cos^3(t)$$

$$y = \operatorname{sen}^3(t)$$

$$0 \leq t \leq 2\pi$$

no ponto $t = \dfrac{\pi}{6}$



Exemplo 5 – Solução

Avaliando das derivadas das componentes

$$\frac{dx}{dt} = \frac{d}{dt}\cos^3(t) = 3\cos^2(t)\frac{d}{dt}\cos(t) = -3\cos^2(t)\,\mathrm{sen}(t)$$

$$\frac{dy}{dt} = \frac{d}{dt}\mathrm{sen}^3(t) = 3\,\mathrm{sen}^2(t)\frac{d}{dt}\mathrm{sen}(t) = 3\,\mathrm{sen}^2(t)\cos(t)$$

Exemplo 5 – Solução

Avaliando a derivada de x no ponto $t = \dfrac{\pi}{6}$

$$\frac{dx}{dt}\left(\frac{\pi}{6}\right) = \left[-3\cos^2(t)\,\mathrm{sen}(t)\right]\Bigg|_{\frac{\pi}{6}} = -3\cos^2\left(\frac{\pi}{6}\right)\mathrm{sen}\left(\frac{\pi}{6}\right)$$

$$= -3\left(\frac{\sqrt{3}}{2}\right)^2\left(\frac{1}{2}\right) = -3\,\frac{3}{4}\,\frac{1}{2} = \frac{-9}{8}$$

Exemplo 5 – Solução

Avaliando a derivada de y no ponto $t = \dfrac{\pi}{6}$

$$\frac{dy}{dt}\left(\frac{\pi}{6}\right) = \left[3\,\mathrm{sen}^2(t)\cos(t)\right]\Big|_{\frac{\pi}{6}} = 3\,\mathrm{sen}^2\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}\right)$$

$$= 3\left(\frac{1}{2}\right)^2\left(\frac{\sqrt{3}}{2}\right) = 3\,\frac{1}{4}\,\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{8}$$

Exemplo 5 – Solução

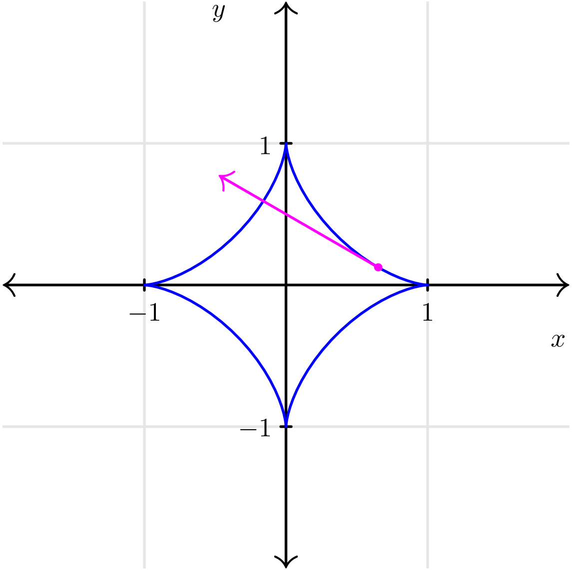
Função vetor tangente

$$v(t) = \frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3\cos^2(t)\,\mathrm{sen}(t) \\ 3\,\mathrm{sen}^2(t)\cos(t) \end{pmatrix}$$

No ponto $t = \dfrac{\pi}{6}$

$$v\left(\frac{\pi}{6}\right) = \begin{pmatrix} \dfrac{-9}{8} \\[2ex] \dfrac{3\sqrt{3}}{8} \end{pmatrix} = \frac{3}{8}\begin{pmatrix} -3 \\ \sqrt{3} \end{pmatrix}$$

Exemplo 5 – Solução



Conteúdo

Lista Mínima

Cálculo Vol. 2 do Thomas 12ª ed. – Seção 11.2

1. Estudar todo o texto da seção
2. Calcule o vetor tangente das funções dos exercícios: 2, 8, 12, 16, 18, 20 (Ignore o enunciado original dos exercícios)

Atenção: A prova é baseada no livro, não nas apresentações