Derivadas Parciais

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Cálculo de Funções de Várias Variáveis – I

Conteúdo

Derivada Ordinária – Uma variável

Variação da função com relação a uma variável

Derivada Ordinária – Uma variável

Variação da função com relação a uma variável

Derivada da função $f\colon R \subset \mathbb{R} \to \mathbb{R}$ no ponto a

$$\frac{df}{dx}(a) = \left.\frac{df}{dx}\right|_{x=a} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

Derivada Parcial

Variação da função com relação a uma das variáveis

considerando as demais fixas

Definição – Derivada Parcial em x

A derivada parcial, com relação a x, da função $f \colon R \subset \mathbb{R}^2 \to \mathbb{R}$,

Definição – Derivada Parcial em x

A derivada parcial, com relação a x, da função $f\colon R \subset \mathbb{R}^2 \to \mathbb{R}$,

$f(x, y)$, no ponto (a, b)

Definição – Derivada Parcial em x

A derivada parcial, com relação a x, da função $f\colon R \subset \mathbb{R}^2 \to \mathbb{R}$,

$f(x, y)$, no ponto (a, b)

$$\frac{\partial f}{\partial x}(a, b) = \left.\frac{\partial f}{\partial x}\right|_{(x,y)=(a,b)} = \lim_{h \to 0} \frac{f(a+h, b) - f(a, b)}{h}$$

Desde que o limite exista

Definição – Derivada Parcial em y

A derivada parcial, com relação a y, da função $f : R \subset \mathbb{R}^2 \to \mathbb{R}$,

Definição – Derivada Parcial em y

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Definição – Derivada Parcial em y

A derivada parcial, com relação a y, da função $f: R \subset \mathbb{R}^2 \to \mathbb{R}$,

$f(x, y)$, no ponto (a, b)

$$\frac{\partial f}{\partial y}(a, b) = \left.\frac{\partial f}{\partial y}\right|_{(x,y)=(a,b)} = \lim_{h \to 0} \frac{f(a, b+h) - f(a, b)}{h}$$

Desde que o limite exista

Notações

$$\frac{\partial f}{\partial y}(x, y) \qquad \frac{\partial f}{\partial y}$$

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$$f_y(x, y) \qquad f_y$$

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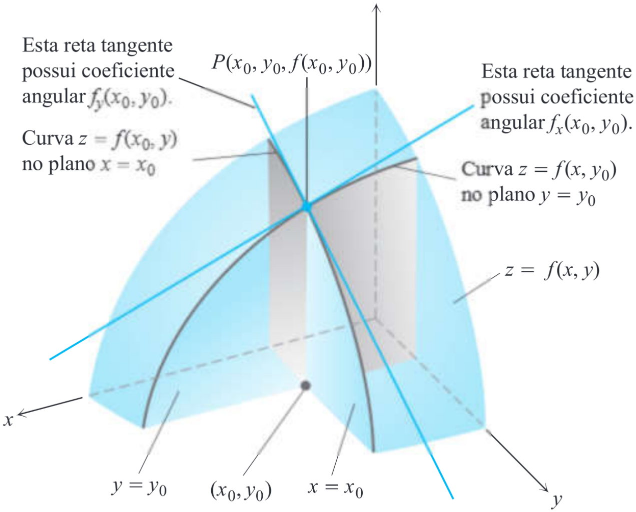
$$f_y(x, y) \qquad f_y$$

$$\partial_y f(x, y) \qquad \partial_y f$$

Retas Tangentes



Retas Tangentes

Temos duas retas tangentes, uma na direção x e uma na direção y

Retas Tangentes

Temos duas retas tangentes, uma na direção x e uma na direção y

Nem sempre o plano gerado por elas é plano tangente da superfície

Conteúdo

Exemplo 1

Encontre os valores de $\dfrac{\partial f}{\partial x}$ e $\dfrac{\partial f}{\partial y}$ no ponto $(4, -5)$ para a função

$$f(x, y) = x^2 + 3xy + y - 1$$

Exemplo 1 – Solução

$$\frac{\partial f}{\partial x}$$

Exemplo 1 – Solução

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(x^2 + 3xy + y - 1\right)$$

Exemplo 1 – Solução

$$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x}\left(x^2 + 3xy + y - 1\right)\\ &= \frac{\partial x^2}{\partial x} + \frac{\partial 3xy}{\partial x} + \frac{\partial y}{\partial x} - \frac{\partial 1}{\partial x}\end{aligned}$$

Exemplo 1 – Solução

$$
\begin{aligned}
\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x}\left(x^2 + 3xy + y - 1\right) \\
&= \frac{\partial x^2}{\partial x} + \frac{\partial 3xy}{\partial x} + \frac{\partial y}{\partial x} - \frac{\partial 1}{\partial x} \\
&= 2x + 3y\frac{\partial x}{\partial x} + 0 - 0
\end{aligned}
$$

Exemplo 1 – Solução

$$
\begin{aligned}
\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x}\left(x^2 + 3xy + y - 1\right) \\
&= \frac{\partial x^2}{\partial x} + \frac{\partial 3xy}{\partial x} + \frac{\partial y}{\partial x} - \frac{\partial 1}{\partial x} \\
&= 2x + 3y\frac{\partial x}{\partial x} + 0 - 0 \\
&= 2x + 3y
\end{aligned}
$$

Exemplo 1 – Solução

$$\frac{\partial f}{\partial x}(4, -5)$$

Exemplo 1 – Solução

$$\frac{\partial f}{\partial x}(4,-5) = (2x+3y)\Big|_{(x,y)=(4,-5)}$$

Exemplo 1 – Solução

$$\frac{\partial f}{\partial x}(4,-5) = (2x+3y)\bigg|_{(x,y)=(4,-5)} = 2 \cdot 4 + 3(-5)$$

Exemplo 1 – Solução

$$\frac{\partial f}{\partial x}(4, -5) = (2x + 3y)\Big|_{(x,y)=(4,-5)} = 2 \cdot 4 + 3(-5) = -7$$

Exemplo 1 – Solução

$$\frac{\partial f}{\partial y}$$

Exemplo 1 – Solução

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(x^2 + 3xy + y - 1\right)$$

Exemplo 1 – Solução

$$
\begin{aligned}
\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y}\left(x^2 + 3xy + y - 1\right) \\
&= \frac{\partial x^2}{\partial y} + \frac{\partial 3xy}{\partial y} + \frac{\partial y}{\partial y} - \frac{\partial 1}{\partial y}
\end{aligned}
$$

Exemplo 1 – Solução

$$\begin{aligned}
\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y}\left(x^2 + 3xy + y - 1\right) \\
&= \frac{\partial x^2}{\partial y} + \frac{\partial 3xy}{\partial y} + \frac{\partial y}{\partial y} - \frac{\partial 1}{\partial y} \\
&= 0 + 3x\frac{\partial y}{\partial y} + \frac{\partial y}{\partial y} - 0
\end{aligned}$$

Exemplo 1 – Solução

$$
\begin{aligned}
\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y}\left(x^2 + 3xy + y - 1\right) \\
&= \frac{\partial x^2}{\partial y} + \frac{\partial 3xy}{\partial y} + \frac{\partial y}{\partial y} - \frac{\partial 1}{\partial y} \\
&= 0 + 3x\frac{\partial y}{\partial y} + \frac{\partial y}{\partial y} - 0 \\
&= 3x + 1
\end{aligned}
$$

Exemplo 1 – Solução

$$
\begin{aligned}
\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y}\left(x^2 + 3xy + y - 1\right) \\
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&= 0 + 3x\frac{\partial y}{\partial y} + \frac{\partial y}{\partial y} - 0 \\
&= 3x + 1
\end{aligned}
$$

Cadê o y?

Exemplo 1 – Solução

$$\frac{\partial f}{\partial y}(4, -5)$$

Exemplo 1 – Solução

$$\frac{\partial f}{\partial y}(4,-5) = (3x+1)\Big|_{(x,y)=(4,-5)}$$

Exemplo 1 – Solução

$$\frac{\partial f}{\partial y}(4,-5) = (3x+1)\bigg|_{(x,y)=(4,-5)} = 3 \cdot 4 + 1$$

Exemplo 1 – Solução

$$\frac{\partial f}{\partial y}(4,-5) = (3x+1)\Big|_{(x,y)=(4,-5)} = 3\cdot 4 + 1 = 13$$

Exemplo 2

Encontre $\dfrac{\partial f}{\partial x}$ e $\dfrac{\partial f}{\partial y}$ para $f(x, y) = y \operatorname{sen}(xy)$

Exemplo 2 – Solução

$$\frac{\partial f}{\partial x}$$

Exemplo 2 – Solução

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\big(y\,\mathrm{sen}(xy)\big)$$

Exemplo 2 – Solução

$$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x}\big(y\,\mathrm{sen}(xy)\big)\\ &= y\frac{\partial}{\partial x}\,\mathrm{sen}(xy)\end{aligned}$$

Exemplo 2 – Solução

$$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x}\big(y\,\mathrm{sen}(xy)\big) \\ &= y\frac{\partial}{\partial x}\,\mathrm{sen}(xy) \\ &= y\cos(xy)\frac{\partial}{\partial x}(xy)\end{aligned}$$

Exemplo 2 – Solução

$$
\begin{aligned}
\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x}\big(y\,\mathrm{sen}(xy)\big) \\
&= y\frac{\partial}{\partial x}\,\mathrm{sen}(xy) \\
&= y\cos(xy)\frac{\partial}{\partial x}(xy) \\
&= y\cos(xy)y
\end{aligned}
$$

Exemplo 2 – Solução

$$\begin{aligned}
\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x}\big(y \operatorname{sen}(xy)\big) \\
&= y \frac{\partial}{\partial x} \operatorname{sen}(xy) \\
&= y \cos(xy) \frac{\partial}{\partial x}(xy) \\
&= y \cos(xy) y \\
&= y^2 \cos(xy)
\end{aligned}$$

Exemplo 2 – Solução

$$\frac{\partial f}{\partial y}$$

Exemplo 2 – Solução

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\big(y\,\mathrm{sen}(xy)\big)$$

Exemplo 2 – Solução

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\big(y \operatorname{sen}(xy)\big)$$

$$= \frac{\partial y}{\partial y} \operatorname{sen}(xy) + y\frac{\partial}{\partial y} \operatorname{sen}(xy)$$

Exemplo 2 – Solução

$$
\begin{aligned}
\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y}\big(y\,\mathrm{sen}(xy)\big) \\
&= \frac{\partial y}{\partial y}\,\mathrm{sen}(xy) + y\frac{\partial}{\partial y}\,\mathrm{sen}(xy) \\
&= \mathrm{sen}(xy) + y\cos(xy)\frac{\partial}{\partial y}(xy)
\end{aligned}
$$

Exemplo 2 – Solução

$$
\begin{aligned}
\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y}\big(y\,\mathrm{sen}(xy)\big) \\
&= \frac{\partial y}{\partial y}\,\mathrm{sen}(xy) + y\frac{\partial}{\partial y}\,\mathrm{sen}(xy) \\
&= \mathrm{sen}(xy) + y\cos(xy)\frac{\partial}{\partial y}(xy) \\
&= \mathrm{sen}(xy) + y\cos(xy)x
\end{aligned}
$$

Exemplo 2 – Solução

$$\begin{aligned}
\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y}\big(y \operatorname{sen}(xy)\big) \\
&= \frac{\partial y}{\partial y} \operatorname{sen}(xy) + y \frac{\partial}{\partial y} \operatorname{sen}(xy) \\
&= \operatorname{sen}(xy) + y \cos(xy) \frac{\partial}{\partial y}(xy) \\
&= \operatorname{sen}(xy) + y \cos(xy) x \\
&= \operatorname{sen}(xy) + xy \cos(xy)
\end{aligned}$$

Exemplo 3

Encontre as funções f_x e f_y sendo que $f(x, y) = \dfrac{2y}{y + \cos(x)}$

Exemplo 3 – Solução

$$\frac{\partial f}{\partial x}$$

$$= 2y\frac{\partial}{\partial x}\big(y + \cos(x)\big)^{-1}$$

$$= 2y(-1)\big(y + \cos(x)\big)^{-2}\ \frac{\partial}{\partial x}\,(y + \cos(x))$$

$$= -2y\big(y + \cos(x)\big)^{-2}\ \big(-\operatorname{sen}(x)\big)$$

Exemplo 3 – Solução

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(\frac{2y}{y+\cos(x)}\right)$$

$$\begin{aligned}
&= 2y\frac{\partial}{\partial x}\big(y+\cos(x)\big)^{-1} \\
&= 2y(-1)\big(y+\cos(x)\big)^{-2}\,\frac{\partial}{\partial x}\,(y+\cos(x)) \\
&= -2y\big(y+\cos(x)\big)^{-2}\,\big(-\operatorname{sen}(x)\big)
\end{aligned}$$

Exemplo 3 – Solução

$$
\begin{aligned}
\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x}\left(\frac{2y}{y+\cos(x)}\right) \\
&= \frac{\partial}{\partial x}\left[2y\big(y+\cos(x)\big)^{-1}\right] \\
&= 2y\frac{\partial}{\partial x}\big(y+\cos(x)\big)^{-1} \\
&= 2y(-1)\big(y+\cos(x)\big)^{-2}\,\frac{\partial}{\partial x}\,(y+\cos(x)) \\
&= -2y\big(y+\cos(x)\big)^{-2}\,\big(-\operatorname{sen}(x)\big)
\end{aligned}
$$

Exemplo 3 – Solução

$$
\begin{aligned}
\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x}\left(\frac{2y}{y+\cos(x)}\right) \\
&= \frac{\partial}{\partial x}\left[2y\big(y+\cos(x)\big)^{-1}\right] \\
&= 2y\frac{\partial}{\partial x}\big(y+\cos(x)\big)^{-1} \\
&= 2y(-1)\big(y+\cos(x)\big)^{-2}\,\frac{\partial}{\partial x}\big(y+\cos(x)\big) \\
&= -2y\big(y+\cos(x)\big)^{-2}\,\big(-\operatorname{sen}(x)\big) \\
&= \frac{2y\operatorname{sen}(x)}{\big(y+\cos(x)\big)^{2}}
\end{aligned}
$$

Exemplo 3 – Solução

$$\frac{\partial f}{\partial y}$$

Exemplo 3 – Solução

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(\frac{2y}{y + \cos(x)}\right)$$

Exemplo 3 – Solução

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(\frac{2y}{y+\cos(x)}\right)$$

$$= 2\,\frac{\dfrac{\partial y}{\partial y}\left(y+\cos(x)\right) - y\dfrac{\partial}{\partial y}\left(y+\cos(x)\right)}{\left(y+\cos(x)\right)^2}$$

Exemplo 3 – Solução

$$\begin{aligned}
\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y}\left(\frac{2y}{y+\cos(x)}\right) \\
&= 2\,\frac{\dfrac{\partial y}{\partial y}\left(y+\cos(x)\right) - y\dfrac{\partial}{\partial y}\left(y+\cos(x)\right)}{\left(y+\cos(x)\right)^2} \\
&= 2\,\frac{\left(y+\cos(x)\right) - y(1+0)}{\left(y+\cos(x)\right)^2}
\end{aligned}$$

Exemplo 3 – Solução

$$\begin{aligned}
\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y}\left(\frac{2y}{y+\cos(x)}\right) \\
&= 2\,\frac{\dfrac{\partial y}{\partial y}\left(y+\cos(x)\right) - y\dfrac{\partial}{\partial y}\left(y+\cos(x)\right)}{\left(y+\cos(x)\right)^2} \\
&= 2\,\frac{\left(y+\cos(x)\right) - y(1+0)}{\left(y+\cos(x)\right)^2} \\
&= \frac{2\cos(x)}{\left(y+\cos(x)\right)^2}
\end{aligned}$$

Exemplo 4

Encontre $\dfrac{\partial z}{\partial x}$ se a equação

$$yz - \ln(z) = x + 1$$

define z como uma função de x e y

Exemplo 4

Encontre $\dfrac{\partial z}{\partial x}$ se a equação

$$yz - \ln(z) = x + 1$$

define z como uma função de x e y

Vamos derivar os dois lados por x, assumindo $z = z(x, y)$

Exemplo 4 – Solução

$$yz - \ln(z) = x + 1$$

Exemplo 4 – Solução

$$yz - \ln(z) = x + 1$$

$$\frac{\partial}{\partial x}\big(yz - \ln(z)\big) = \frac{\partial}{\partial x}(x + 1)$$

Exemplo 4 – Solução

$$yz - \ln(z) = x + 1$$

$$\frac{\partial}{\partial x}\big(yz - \ln(z)\big) = \frac{\partial}{\partial x}\,(x + 1)$$

$$\frac{\partial}{\partial x}(yz) - \frac{\partial}{\partial x}\big(\,\ln(z)\big) = 1$$

Exemplo 4 – Solução

$$yz - \ln(z) = x + 1$$

$$\frac{\partial}{\partial x}\big(yz - \ln(z)\big) = \frac{\partial}{\partial x}(x + 1)$$

$$\frac{\partial}{\partial x}(yz) - \frac{\partial}{\partial x}\big(\ln(z)\big) = 1$$

$$y\frac{\partial z}{\partial x} - \frac{1}{z}\frac{\partial z}{\partial x} = 1$$

Exemplo 4 – Solução

$$yz - \ln(z) = x + 1$$

$$\frac{\partial}{\partial x}\big(yz - \ln(z)\big) = \frac{\partial}{\partial x}(x+1)$$

$$\frac{\partial}{\partial x}(yz) - \frac{\partial}{\partial x}\big(\ln(z)\big) = 1$$

$$y\frac{\partial z}{\partial x} - \frac{1}{z}\frac{\partial z}{\partial x} = 1$$

$$\frac{\partial z}{\partial x}\left(y - \frac{1}{z}\right) = 1$$

Exemplo 4 – Solução

$$yz - \ln(z) = x + 1$$

$$\frac{\partial}{\partial x}\big(yz - \ln(z)\big) = \frac{\partial}{\partial x}(x + 1)$$

$$\frac{\partial}{\partial x}(yz) - \frac{\partial}{\partial x}\big(\ln(z)\big) = 1$$

$$y\frac{\partial z}{\partial x} - \frac{1}{z}\frac{\partial z}{\partial x} = 1$$

$$\frac{\partial z}{\partial x}\left(y - \frac{1}{z}\right) = 1$$

$$\frac{\partial z}{\partial x} = \left(y - \frac{1}{z}\right)^{-1}$$

Exemplo 4 – Solução

$$yz - \ln(z) = x + 1$$

$$\frac{\partial}{\partial x}\big(yz - \ln(z)\big) = \frac{\partial}{\partial x}(x+1)$$

$$\frac{\partial}{\partial x}(yz) - \frac{\partial}{\partial x}\big(\ln(z)\big) = 1$$

$$y\frac{\partial z}{\partial x} - \frac{1}{z}\frac{\partial z}{\partial x} = 1$$

$$\frac{\partial z}{\partial x}\left(y - \frac{1}{z}\right) = 1$$

$$\frac{\partial z}{\partial x} = \left(y - \frac{1}{z}\right)^{-1}$$

$$= \left(\frac{yz - 1}{z}\right)^{-1}$$

Exemplo 4 – Solução

$$yz - \ln(z) = x + 1$$

$$\frac{\partial}{\partial x}\big(yz - \ln(z)\big) = \frac{\partial}{\partial x}(x+1)$$

$$\frac{\partial}{\partial x}(yz) - \frac{\partial}{\partial x}\big(\ln(z)\big) = 1$$

$$y\frac{\partial z}{\partial x} - \frac{1}{z}\frac{\partial z}{\partial x} = 1$$

$$\frac{\partial z}{\partial x}\left(y - \frac{1}{z}\right) = 1$$

$$\frac{\partial z}{\partial x} = \left(y - \frac{1}{z}\right)^{-1} = \left(\frac{yz-1}{z}\right)^{-1} = \frac{z}{yz-1}$$

Exemplo 5

Calcule as derivadas parciais de primeira ordem da função

$$f(x,y) = \operatorname{sen}\left(x^2 y\right) + e^{xy^2}$$

Exemplo 5 – Solução

$$\frac{\partial f}{\partial x}$$

Exemplo 5 – Solução

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(\operatorname{sen}\left(x^2 y\right) + e^{xy^2}\right)$$

Exemplo 5 – Solução

$$
\begin{aligned}
\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x}\left(\operatorname{sen}\left(x^2 y\right) + e^{xy^2}\right) \\
&= \frac{\partial}{\partial x}\left(\operatorname{sen}\left(x^2 y\right)\right) + \frac{\partial}{\partial x}\left(e^{xy^2}\right)
\end{aligned}
$$

Exemplo 5 – Solução

$$
\begin{aligned}
\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x}\left(\operatorname{sen}\left(x^2 y\right) + e^{xy^2}\right) \\
&= \frac{\partial}{\partial x}\left(\operatorname{sen}\left(x^2 y\right)\right) + \frac{\partial}{\partial x}\left(e^{xy^2}\right) \\
&= \cos\left(x^2 y\right)\frac{\partial}{\partial x}\left(x^2 y\right) + e^{xy^2}\frac{\partial}{\partial x}\left(xy^2\right)
\end{aligned}
$$

Exemplo 5 – Solução

$$
\begin{aligned}
\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x}\left(\operatorname{sen}\left(x^2 y\right) + e^{xy^2}\right) \\
&= \frac{\partial}{\partial x}\left(\operatorname{sen}\left(x^2 y\right)\right) + \frac{\partial}{\partial x}\left(e^{xy^2}\right) \\
&= \cos\left(x^2 y\right)\frac{\partial}{\partial x}\left(x^2 y\right) + e^{xy^2}\frac{\partial}{\partial x}\left(xy^2\right) \\
&= \cos\left(x^2 y\right) 2xy + e^{xy^2} y^2
\end{aligned}
$$

Exemplo 5 – Solução

$$
\begin{aligned}
\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x}\left(\operatorname{sen}\left(x^2 y\right) + e^{xy^2}\right) \\
&= \frac{\partial}{\partial x}\left(\operatorname{sen}\left(x^2 y\right)\right) + \frac{\partial}{\partial x}\left(e^{xy^2}\right) \\
&= \cos\left(x^2 y\right)\frac{\partial}{\partial x}\left(x^2 y\right) + e^{xy^2}\frac{\partial}{\partial x}\left(xy^2\right) \\
&= \cos\left(x^2 y\right) 2xy + e^{xy^2} y^2 \\
&= 2xy\cos\left(x^2 y\right) + y^2 e^{xy^2}
\end{aligned}
$$

Exemplo 5 – Solução

$$\frac{\partial f}{\partial y}$$

Exemplo 5 – Solução

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(\operatorname{sen}\left(x^2 y\right) + e^{xy^2}\right)$$

Exemplo 5 – Solução

$$\begin{aligned}\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y}\left(\operatorname{sen}\left(x^2 y\right) + e^{xy^2}\right)\\ &= \frac{\partial}{\partial y}\left(\operatorname{sen}\left(x^2 y\right)\right) + \frac{\partial}{\partial y}\left(e^{xy^2}\right)\end{aligned}$$

Exemplo 5 – Solução

$$
\begin{aligned}
\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y}\left(\operatorname{sen}\left(x^2 y\right) + e^{xy^2}\right) \\
&= \frac{\partial}{\partial y}\left(\operatorname{sen}\left(x^2 y\right)\right) + \frac{\partial}{\partial y}\left(e^{xy^2}\right) \\
&= \cos\left(x^2 y\right)\frac{\partial}{\partial y}\left(x^2 y\right) + e^{xy^2}\frac{\partial}{\partial y}\left(xy^2\right)
\end{aligned}
$$

Exemplo 5 – Solução

$$\begin{aligned}
\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y}\left(\operatorname{sen}\left(x^2 y\right) + e^{xy^2}\right) \\
&= \frac{\partial}{\partial y}\left(\operatorname{sen}\left(x^2 y\right)\right) + \frac{\partial}{\partial y}\left(e^{xy^2}\right) \\
&= \cos\left(x^2 y\right)\frac{\partial}{\partial y}\left(x^2 y\right) + e^{xy^2}\frac{\partial}{\partial y}\left(xy^2\right) \\
&= \cos\left(x^2 y\right) x^2 + e^{xy^2} 2xy
\end{aligned}$$

Exemplo 5 – Solução

$$
\begin{aligned}
\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y}\left(\operatorname{sen}\left(x^2 y\right) + e^{xy^2}\right) \\
&= \frac{\partial}{\partial y}\left(\operatorname{sen}\left(x^2 y\right)\right) + \frac{\partial}{\partial y}\left(e^{xy^2}\right) \\
&= \cos\left(x^2 y\right) \frac{\partial}{\partial y}\left(x^2 y\right) + e^{xy^2} \frac{\partial}{\partial y}\left(xy^2\right) \\
&= \cos\left(x^2 y\right) x^2 + e^{xy^2} 2xy \\
&= x^2 \cos\left(x^2 y\right) + 2xye^{xy^2}
\end{aligned}
$$

Exemplo 6

Calcule as derivadas parciais de primeira ordem da função

$$f(x,y) = \operatorname{arctg}\left(\frac{x}{y}\right)$$

Exemplo 6

Calcule as derivadas parciais de primeira ordem da função

$$f(x,y) = \operatorname{arctg}\left(\frac{x}{y}\right)$$

$$\frac{d}{du}\operatorname{arctg}(u) = \operatorname{arctg}'(u) = \frac{1}{1+u^2}$$

Exemplo 6 – Solução

$$
\begin{aligned}
\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x}\left(\operatorname{arctg}\left(\frac{x}{y}\right)\right) \\
&= \operatorname{arctg}'\left(\frac{x}{y}\right)\frac{\partial}{\partial x}\left(\frac{x}{y}\right) \\
&= \frac{1}{1+\left(\frac{x}{y}\right)^2}\frac{\partial}{\partial x}\left(\frac{x}{y}\right) \\
&= \frac{1}{1+\left(\frac{x}{y}\right)^2}\left(\frac{1}{y}\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial f}{\partial x} &= \frac{1}{1+\frac{x^2}{y^2}}\left(\frac{1}{y}\right) \\
&= \frac{1}{\frac{y^2+x^2}{y^2}}\left(\frac{1}{y}\right) \\
&= \frac{y^2}{y^2+x^2}\left(\frac{1}{y}\right) \\
&= \frac{y}{x^2+y^2}
\end{aligned}
$$

Exemplo 6 – Solução

$$
\begin{aligned}
\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y}\left(\operatorname{arctg}\left(\frac{x}{y}\right)\right) \\
&= \operatorname{arctg}'\left(\frac{x}{y}\right)\frac{\partial}{\partial y}\left(\frac{x}{y}\right) \\
&= \frac{1}{1+\left(\frac{x}{y}\right)^2}\frac{\partial}{\partial y}\left(xy^{-1}\right) \\
&= \frac{1}{1+\left(\frac{x}{y}\right)^2}\left(-xy^{-2}\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial f}{\partial x} &= \frac{1}{1+\frac{x^2}{y^2}}\left(-xy^{-2}\right) \\
&= \frac{1}{\frac{y^2+x^2}{y^2}}\left(-xy^{-2}\right) \\
&= \frac{y^2}{y^2+x^2}\left(-xy^{-2}\right) \\
&= \frac{-x}{x^2+y^2}
\end{aligned}
$$

Conteúdo

Lista Mínima

Cálculo Vol. 2 do Thomas 12ª ed. – Seção 14.3

1. Estudar o texto da seção
2. Resolver os exercícios: 4, 10, 37-40

Atenção: A prova é baseada no livro, não nas apresentações