Plano Complexo

Luis Alberto D'Afonseca

Cálculo de Funções de Várias Variáveis – I

https://material-didatico.github.io/pages/cfvv1

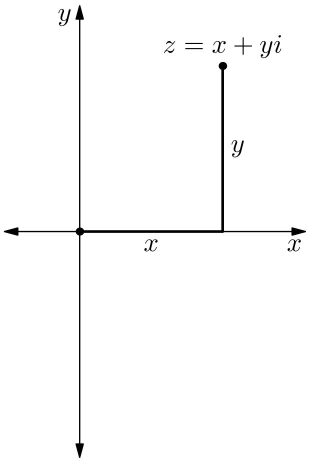
11 de março de 2026

Conteúdo

Plano Complexo

$$z = x + yi \quad \in \mathbb{C}$$

$$(x, y) \quad \in \mathbb{R}^2$$



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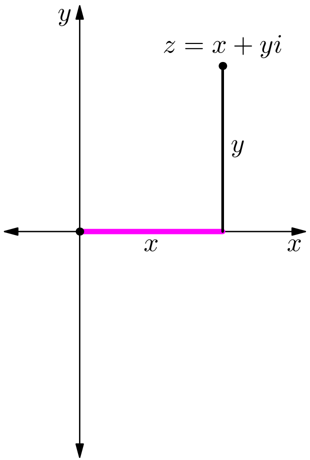
Parte real $\quad \mathrm{Re}(z) = x$



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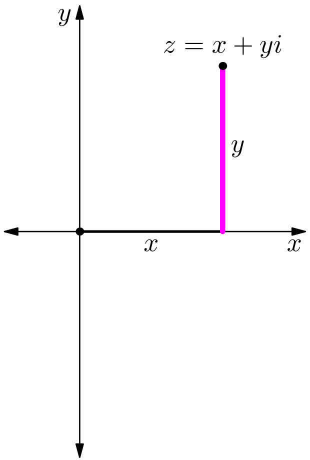

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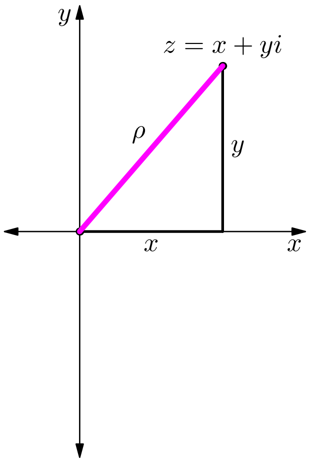
Módulo



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Parte real	$\mathrm{Re}(z) = x$
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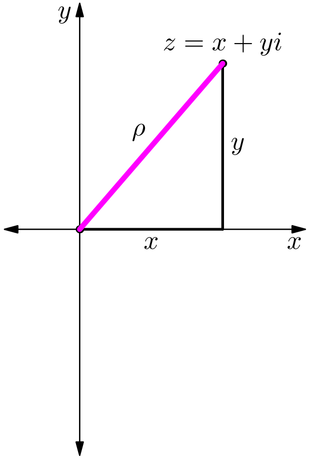
Módulo	$\lvert z \rvert$



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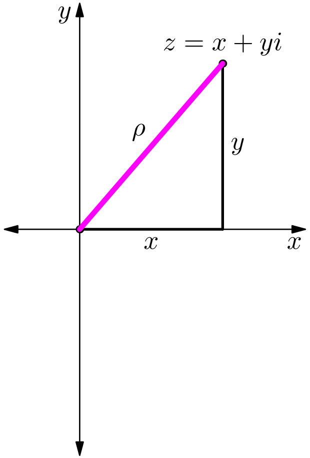
Módulo $\quad |z| = \sqrt{z\bar{z}}$



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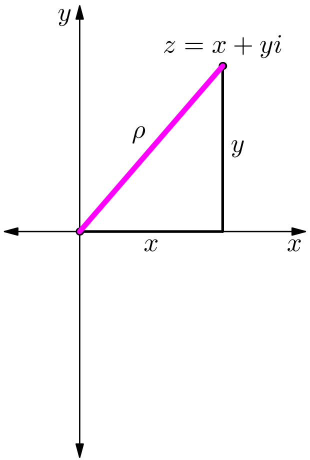

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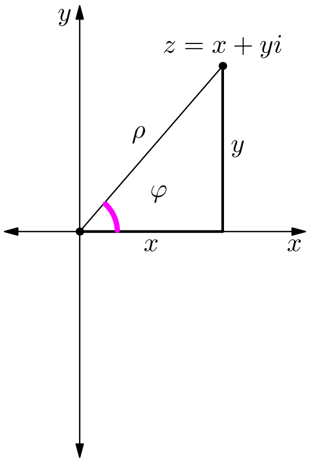
Argumento	



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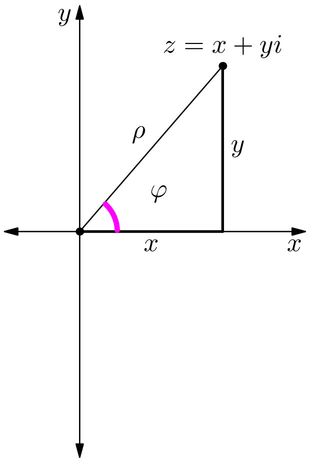
Argumento $\quad \arg(z)$



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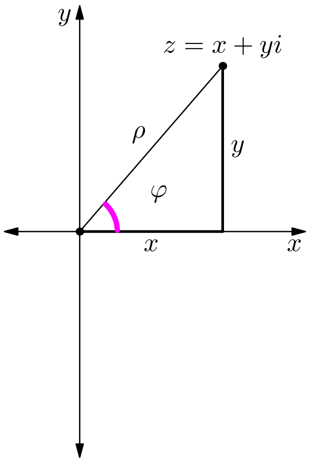

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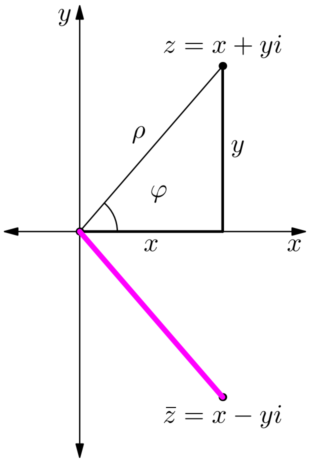
Conjugado	$\bar{z} = x - yi$



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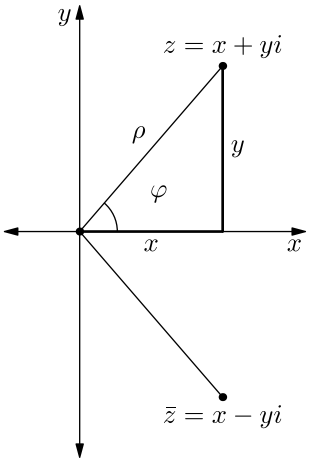

Exemplo 1

Dado $z = \dfrac{\left(1 - \sqrt{3}\right) + \left(1 + \sqrt{3}\right) i}{2 + 2i}$

calcule

a) a parte real de z

b) a parte imaginária de z

c) o módulo de z

d) o argumento de z

Exemplo 1 – Avaliando z

$$z = \frac{\left(1 - \sqrt{3}\right) + \left(1 + \sqrt{3}\right)\, i}{2 + 2i}$$

Exemplo 1 – Avaliando z

$$z = \frac{\left(1-\sqrt{3}\right) + \left(1+\sqrt{3}\right)\, i}{2+2i} \quad = \quad \frac{\left[\left(1-\sqrt{3}\right) + \left(1+\sqrt{3}\right)\, i\right](2-2i)}{(2+2i)(2-2i)}$$

Exemplo 1 – Avaliando z

$$z = \frac{\left(1-\sqrt{3}\right)+\left(1+\sqrt{3}\right)\,i}{2+2i} \quad = \quad \frac{\left[\left(1-\sqrt{3}\right)+\left(1+\sqrt{3}\right)\,i\right](2-2i)}{(2+2i)(2-2i)}$$

$$= \frac{\left(1-\sqrt{3}\right)(2-2i)+\left(1+\sqrt{3}\right)\,i(2-2i)}{2^2+2^2}$$

Exemplo 1 – Avaliando z

$$z = \frac{\left(1-\sqrt{3}\right)+\left(1+\sqrt{3}\right)\,i}{2+2i} \quad = \quad \frac{\left[\left(1-\sqrt{3}\right)+\left(1+\sqrt{3}\right)\,i\right]\,(2-2i)}{(2+2i)(2-2i)}$$

$$= \frac{\left(1-\sqrt{3}\right)(2-2i)+\left(1+\sqrt{3}\right)\,i(2-2i)}{2^2+2^2}$$

$$= \frac{\left(1-\sqrt{3}\right)(2-2i)+\left(1+\sqrt{3}\right)(2i+2)}{8}$$

Exemplo 1 – Avaliando z

$$
\begin{aligned}
z = \frac{\left(1-\sqrt{3}\right) + \left(1+\sqrt{3}\right)\, i}{2+2i} \quad &= \quad \frac{\left[\left(1-\sqrt{3}\right) + \left(1+\sqrt{3}\right)\, i\right]\,(2-2i)}{(2+2i)(2-2i)} \\
&= \frac{\left(1-\sqrt{3}\right)(2-2i) + \left(1+\sqrt{3}\right) i(2-2i)}{2^2+2^2} \\
&= \frac{\left(1-\sqrt{3}\right)(2-2i) + \left(1+\sqrt{3}\right)(2i+2)}{8} \\
&= \frac{2-2i-2\sqrt{3}+2\sqrt{3}i+2i+2+2\sqrt{3}i+2\sqrt{3}}{8}
\end{aligned}
$$

Exemplo 1 – Avaliando z

$$
\begin{aligned}
z = \frac{\left(1-\sqrt{3}\right)+\left(1+\sqrt{3}\right)\, i}{2+2i} \quad &= \quad \frac{\left[\left(1-\sqrt{3}\right)+\left(1+\sqrt{3}\right)\, i\right]\,(2-2i)}{(2+2i)(2-2i)} \\
&= \frac{\left(1-\sqrt{3}\right)(2-2i)+\left(1+\sqrt{3}\right)\, i(2-2i)}{2^2+2^2} \\
&= \frac{\left(1-\sqrt{3}\right)(2-2i)+\left(1+\sqrt{3}\right)(2i+2)}{8} \\
&= \frac{2-2i-2\sqrt{3}+2\sqrt{3}i+2i+2+2\sqrt{3}i+2\sqrt{3}}{8} \\
&= \frac{4+4\sqrt{3}i}{8}
\end{aligned}
$$

Exemplo 1 – Avaliando z

$$z = \frac{\left(1-\sqrt{3}\right)+\left(1+\sqrt{3}\right)\,i}{2+2i} = \frac{\left[\left(1-\sqrt{3}\right)+\left(1+\sqrt{3}\right)\,i\right](2-2i)}{(2+2i)(2-2i)}$$

$$= \frac{\left(1-\sqrt{3}\right)(2-2i)+\left(1+\sqrt{3}\right)i(2-2i)}{2^2+2^2}$$

$$= \frac{\left(1-\sqrt{3}\right)(2-2i)+\left(1+\sqrt{3}\right)(2i+2)}{8}$$

$$= \frac{2-2i-2\sqrt{3}+2\sqrt{3}i+2i+2+2\sqrt{3}i+2\sqrt{3}}{8}$$

$$= \frac{4+4\sqrt{3}i}{8} = \frac{1}{2}+\frac{\sqrt{3}}{2}i$$

Exemplo 1 – Partes real e imaginária de z

$$z = \frac{1}{2} + \frac{\sqrt{3}}{2} i$$

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Parte real de z

$$\mathrm{Re}(z) = \frac{1}{2}$$

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Parte real de z

$$\mathrm{Re}(z) = \frac{1}{2}$$

Parte imaginária de z

$$\mathrm{Im}(z) = \frac{\sqrt{3}}{2}$$

Exemplo 1 – Módulo de z

$$|z|$$

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Exemplo 1 – Módulo de z

$$|z| = \sqrt{z\bar{z}} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{\frac{1+3}{4}}$$

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Exemplo 1 – Módulo de z

$$|z| = \sqrt{z\bar{z}} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{\frac{1+3}{4}} = \sqrt{1} = 1$$

Exemplo 1 – Argumento de z

$$\varphi = \arg(z)$$

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$$\operatorname{sen}(\varphi)$$

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$$\varphi = \arg(z)$$

$$\operatorname{sen}(\varphi) = \frac{\operatorname{Im}(z)}{|z|}$$

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$$\operatorname{sen}(\varphi) = \frac{\operatorname{Im}(z)}{|z|} = \frac{\sqrt{3}}{2}$$

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$$\cos(\varphi)$$

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$$\mathrm{sen}(\varphi) = \frac{\mathrm{Im}(z)}{|z|} = \frac{\sqrt{3}}{2}$$

$$\cos(\varphi) = \frac{\mathrm{Re}(z)}{|z|}$$

Exemplo 1 – Argumento de z

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$$\operatorname{sen}(\varphi) = \frac{\operatorname{Im}(z)}{|z|} = \frac{\sqrt{3}}{2}$$

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$$\arg(z)$$

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$$\cos(\varphi) = \frac{\operatorname{Re}(z)}{|z|} = \frac{1}{2}$$

$$\arg(z) = 60^\circ$$

Exemplo 1 – Argumento de z

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$$\operatorname{sen}(\varphi) = \frac{\operatorname{Im}(z)}{|z|} = \frac{\sqrt{3}}{2}$$

$$\cos(\varphi) = \frac{\operatorname{Re}(z)}{|z|} = \frac{1}{2}$$

$$\arg(z) = 60^\circ = \frac{\pi}{3}\mathrm{rad}$$

Conteúdo

Lista Mínima

Atenção: A prova é baseada no livro, não nas apresentações