

Integração por Substituição Trigonométrica

3 – Substituição pela secante

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Integração e Séries



<https://material-didatico.github.io/pages/is>

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Substituições Trigonômétricas

Expressão	Substituição	Intervalo	Identidade
$\sqrt{a^2 - x^2}$	$x = a \operatorname{sen}(\theta)$	$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$	$1 - \operatorname{sen}^2(\theta) = \cos^2(\theta)$
$\sqrt{a^2 + x^2}$	$x = a \operatorname{tg}(\theta)$	$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$1 + \operatorname{tg}^2(\theta) = \sec^2(\theta)$
$\sqrt{x^2 - a^2}$	$x = a \sec(\theta)$	$0 \leq \theta < \frac{\pi}{2} \text{ ou } \pi \leq \theta < \frac{3\pi}{2}$	$\sec^2(\theta) - 1 = \operatorname{tg}^2(\theta)$

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Encontre $\int \frac{dx}{\sqrt{x^2 - a^2}}$ com $a > 0$

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Substituição

$$x = a \sec(\theta) \quad dx = a \sec(\theta) \operatorname{tg}(\theta) d\theta$$

onde $0 < \theta < \frac{\pi}{2}$ ou $\pi < \theta < \frac{3\pi}{2}$

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$$F = \int \frac{a \sec(\theta) \operatorname{tg}(\theta)}{\sqrt{a^2 \sec^2(\theta) - a^2}} d\theta$$

Exemplo 1

$$F = \int \frac{dx}{\sqrt{x^2 - a^2}}$$

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Exemplo 1

Usando $\sec^2(\theta) - 1 = \operatorname{tg}^2(\theta)$

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$$F = \int \frac{\sec(\theta) \operatorname{tg}(\theta)}{\sqrt{\operatorname{tg}^2(\theta)}} d\theta$$

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Exemplo 1

$$F = \int \sec(\theta) d\theta$$

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$$\begin{aligned} F &= \int \sec(\theta) d\theta \\ &= \int \sec(\theta) \frac{\sec(\theta) + \operatorname{tg}(\theta)}{\sec(\theta) + \operatorname{tg}(\theta)} d\theta \end{aligned}$$

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Substituição $u = \sec(\theta) + \operatorname{tg}(\theta)$ $du = (\sec(\theta) \operatorname{tg}(\theta) + \sec^2(\theta)) d\theta$

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Substituição $u = \sec(\theta) + \operatorname{tg}(\theta)$ $du = (\sec(\theta) \operatorname{tg}(\theta) + \sec^2(\theta)) d\theta$

$$F = \int \frac{du}{u} = \ln|u| + C$$

Exemplo 1

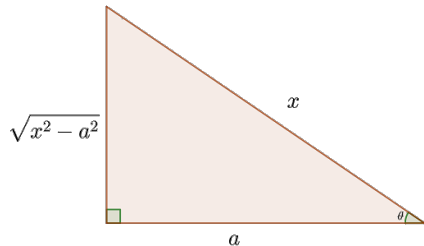
$$\begin{aligned} F &= \int \sec(\theta) d\theta \\ &= \int \sec(\theta) \frac{\sec(\theta) + \operatorname{tg}(\theta)}{\sec(\theta) + \operatorname{tg}(\theta)} d\theta \\ &= \int \frac{\sec^2(\theta) + \sec(\theta) \operatorname{tg}(\theta)}{\sec(\theta) + \operatorname{tg}(\theta)} d\theta \end{aligned}$$

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$$F = \int \frac{du}{u} = \ln|u| + C = \ln|\sec(\theta) + \operatorname{tg}(\theta)| + C$$

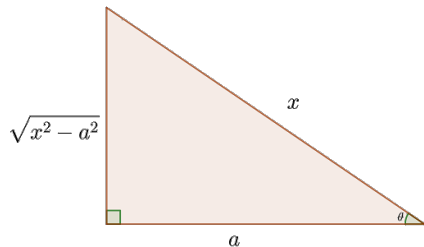
Exemplo 1

$$x = a \sec(\theta)$$



Exemplo 1

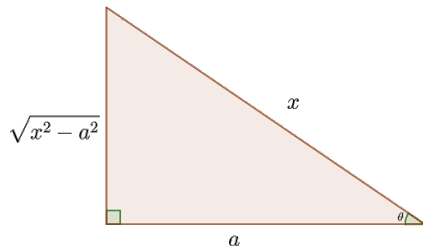
$$x = a \sec(\theta) \quad \Rightarrow \quad \sec(\theta) = \frac{\text{hipotenusa}}{\text{cateto adjacente}} = \frac{x}{a}$$



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$$\text{tg}(\theta) = \frac{\text{cateto oposto}}{\text{cateto adjacente}} = \frac{\sqrt{x^2 - a^2}}{a}$$

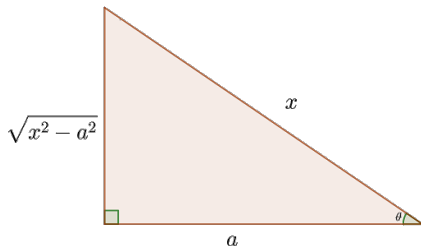


Exemplo 1

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$$F = \int \frac{dx}{\sqrt{x^2 - a^2}}$$



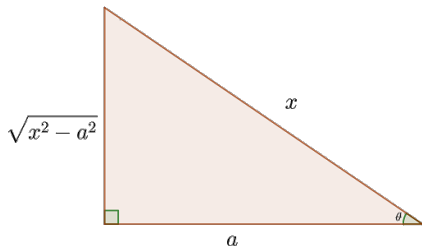
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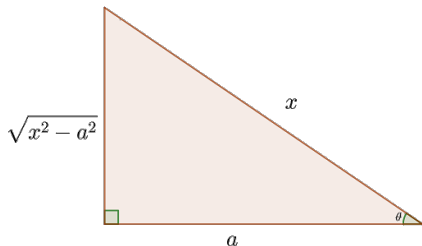
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$$= \ln|\sec(\theta) + \text{tg}(\theta)| + C$$

$$= \ln \left| \frac{x}{a} + \frac{\sqrt{x^2 - a^2}}{a} \right| + C$$



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Estudar a Seção 4.4 da Apostila

Exercícios:

Atenção: A prova é baseada no livro, não nas apresentações