

Forma Polar Números Complexos

Luis Alberto D'Afonseca

Cálculo de Funções de Várias Variáveis – I



<https://material-didatico.github.io/cfvv1>

Conteúdo

Forma Polar ou Trigonométrica de um Número Complexo

Multiplicação

Divisão

Lista Mínima

Plano Complexo

$$z = x + yi \quad \in \mathbb{C}$$

$$(x, y) \quad \in \mathbb{R}^2$$

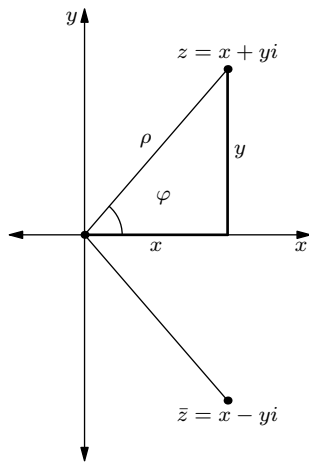
$$\text{Parte real} \quad \operatorname{Re}(z) = x$$

$$\text{Parte imag} \quad \operatorname{Im}(z) = y$$

$$\text{Módulo} \quad \rho = |z| = \sqrt{z\bar{z}} = \sqrt{x^2 + y^2}$$

$$\text{Argumento} \quad \varphi = \arg(z) = \operatorname{arctg}\left(\frac{y}{x}\right)$$

$$\text{Conjugado} \quad \bar{z} = x - yi$$



Forma Polar ou Trigonométrica de um Número Complexo

Usando

$$x = \rho \cos(\varphi) \quad \text{e} \quad y = \rho \operatorname{sen}(\varphi)$$

Podemos escrever o número complexo $z = x + yi$ como

$$z = \rho (\cos(\varphi) + i \operatorname{sen}(\varphi))$$

Abreviação

$$z = \rho [\cos (()) + i \operatorname{sen} (())] \varphi)$$

Notação de Steinmetz (circuitos de corrente alternada)

$$z = \rho \angle \varphi$$

Exemplo 1

Escreva os números complexos na sua forma polar

1. $z_1 = \sqrt{3} + i$

2. $z_2 = -2 + 2\sqrt{3}i$

Exemplo 1 – Solução – 1

$$z_1 = \sqrt{3} + i \qquad a = \operatorname{Re}(z) = \sqrt{3} \qquad b = \operatorname{Im}(z) = 1$$

$$\rho = |z_1| = \sqrt{a^2 + b^2} = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3 + 1} = \sqrt{4} = 2$$

$$\operatorname{sen}(\varphi) = \frac{b}{\rho} = \frac{1}{2} \qquad \cos(\varphi) = \frac{a}{\rho} = \frac{\sqrt{3}}{2}$$

$$\varphi = \arg(z_1) = 30^\circ = \frac{\pi}{6} \text{rad}$$

$$z_1 = \rho \left(\cos(\varphi) + i \operatorname{sen}(\varphi) \right) = 2 \left(\cos\left(\frac{\pi}{6}\right) + i \operatorname{sen}\left(\frac{\pi}{6}\right) \right)$$

Exemplo 1 – Solução – 2

$$z_2 = -2 + 2\sqrt{3}i \qquad a = \operatorname{Re}(z_2) = -2 \qquad b = \operatorname{Im}(z_2) = 2\sqrt{3}$$

$$\rho = |z_2| = \sqrt{a^2 + b^2} = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} = \sqrt{3 + 1} = \sqrt{4} = 2$$

$$\operatorname{sen}(\varphi) = \frac{\operatorname{Im}(z_2)}{|z_2|} = \frac{b}{\rho} = \frac{1}{2} \qquad \cos(\varphi) = \frac{\operatorname{Re}(z_2)}{|z_2|} = \frac{a}{\rho} = \frac{\sqrt{3}}{2}$$

$$\varphi = \arg(z_2) = 30^\circ = \frac{\pi}{6} \text{rad}$$

$$z_1 = \rho \left(\cos(\varphi) + i \operatorname{sen}(\varphi) \right) = 2 \left(\cos\left(\frac{\pi}{6}\right) + i \operatorname{sen}\left(\frac{\pi}{6}\right) \right)$$

Conteúdo

Forma Polar ou Trigonométrica de um Número Complexo

Multiplicação

Divisão

Lista Mínima

Justificativa

$$\begin{aligned}z_1 z_2 &= \rho_1 [\cos(\varphi_1) + i \operatorname{sen}(\varphi_1)] \rho_2 [\cos(\varphi_2) + i \operatorname{sen}(\varphi_2)] \\&= \rho_1 \rho_2 [\cos(\varphi_1) + i \operatorname{sen}(\varphi_1)] [\cos(\varphi_2) + i \operatorname{sen}(\varphi_2)] \\&= \rho_1 \rho_2 [\cos(\varphi_1) \cos(\varphi_2) + i \cos(\varphi_1) \operatorname{sen}(\varphi_2) \\&\quad + i \operatorname{sen}(\varphi_1) \cos(\varphi_2) + i^2 \operatorname{sen}(\varphi_1) \operatorname{sen}(\varphi_2)] \\&= \rho_1 \rho_2 [\cos(\varphi_1) \cos(\varphi_2) - \operatorname{sen}(\varphi_1) \operatorname{sen}(\varphi_2) \\&\quad + i (\cos(\varphi_1) \operatorname{sen}(\varphi_2) + \operatorname{sen}(\varphi_1) \cos(\varphi_2))] \\&= \rho_1 \rho_2 [\cos(\varphi_1 + \varphi_2) + i \operatorname{sen}(\varphi_1 + \varphi_2)]\end{aligned}$$

Multiplicação

$$\begin{aligned} z_1 z_2 \cdots z_n &= \rho_1 [\cos(\varphi_1) + i \operatorname{sen}(\varphi_1)] \\ &\quad \rho_2 [\cos(\varphi_2) + i \operatorname{sen}(\varphi_2)] \\ &\quad \vdots \\ &\quad \rho_n [\cos(\varphi_n) + i \operatorname{sen}(\varphi_n)] \\ &= \rho_1 \rho_2 \cdots \rho_n [\cos(\varphi_1 + \varphi_2 + \cdots + \varphi_n) + i \operatorname{sen}(\varphi_1 + \varphi_2 + \cdots + \varphi_n)] \end{aligned}$$

Exemplo 2

Calcule o produto dos números complexos

$$z_1 = 2 \left(\cos \left(\frac{\pi}{6} \right) + i \operatorname{sen} \left(\frac{\pi}{6} \right) \right)$$

$$z_2 = 5 \left(\cos \left(\frac{\pi}{3} \right) + i \operatorname{sen} \left(\frac{\pi}{3} \right) \right)$$

Exemplo 2 – Solução

$$z_1 = 2 \left(\cos \left(\frac{\pi}{6} \right) + i \operatorname{sen} \left(\frac{\pi}{6} \right) \right) \text{ e } z_2 = 5 \left(\cos \left(\frac{\pi}{3} \right) + i \operatorname{sen} \left(\frac{\pi}{3} \right) \right)$$

$$\rho_1 = |z_1| = 2 \qquad \varphi_1 = \arg(z_1) = \frac{\pi}{6}$$

$$\rho_2 = |z_2| = 5 \qquad \varphi_2 = \arg(z_2) = \frac{\pi}{3}$$

$$\rho = |z_1 z_2| = \rho_1 \rho_2 = 2 \times 5 = 10$$

$$\varphi = \arg(z_1 z_2) = \varphi_1 + \varphi_2 = \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi + 2\pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2}$$

$$z = z_1 z_2 = \rho (\cos(\varphi) + i \operatorname{sen}(\varphi)) = 10 \left(\cos \left(\frac{\pi}{2} \right) + i \operatorname{sen} \left(\frac{\pi}{2} \right) \right) = 10i$$

Exemplo 3

Calcule o produto dos números complexos

$$z_1 = 3 \left(\cos \left(\frac{2\pi}{3} \right) + i \operatorname{sen} \left(\frac{2\pi}{3} \right) \right)$$

$$z_2 = \sqrt{2} \left(\cos \left(\frac{\pi}{2} \right) + i \operatorname{sen} \left(\frac{\pi}{2} \right) \right)$$

Exemplo 3 – Solução

$$z_1 = 3 \left(\cos \left(\frac{2\pi}{3} \right) + i \operatorname{sen} \left(\frac{2\pi}{3} \right) \right) \text{ e } z_2 = \sqrt{2} \left(\cos \left(\frac{\pi}{2} \right) + i \operatorname{sen} \left(\frac{\pi}{2} \right) \right)$$

$$\rho_1 = 3 = |z_1| \qquad \varphi_1 = \arg(z_1) = \frac{2\pi}{3}$$

$$\rho_2 |z_2| = \sqrt{2} \qquad \varphi_2 = \arg(z_2) = \frac{\pi}{2}$$

$$\rho = \rho_1 \rho_2 = 3\sqrt{2}$$

$$\varphi = \varphi_1 + \varphi_2 = \frac{2\pi}{3} + \frac{\pi}{2} = \frac{4\pi + 3\pi}{6} = \frac{7\pi}{6}$$

$$z = z_1 z_2 = \rho (\cos(\varphi) + i \operatorname{sen}(\varphi)) = 3\sqrt{2} \left(\cos \left(\frac{7\pi}{6} \right) + i \operatorname{sen} \left(\frac{7\pi}{6} \right) \right)$$

Conteúdo

Forma Polar ou Trigonométrica de um Número Complexo

Multiplicação

Divisão

Lista Mínima

Divisão

A divisão de

$$z_1 = \rho_1 (\cos(\varphi_1) + i \operatorname{sen}(\varphi_1))$$

por

$$z_2 = \rho_2 (\cos(\varphi_2) + i \operatorname{sen}(\varphi_2))$$

é

$$\frac{z_1}{z_2} = \frac{\rho_1}{\rho_2} [\cos(\varphi_1 - \varphi_2) + i \operatorname{sen}(\varphi_1 - \varphi_2)]$$

Exemplo 4

Divida

$$z_1 = 6 \left(\cos \left(\frac{2\pi}{3} \right) + i \operatorname{sen} \left(\frac{2\pi}{3} \right) \right)$$

por

$$z_2 = 3 \left(\cos \left(\frac{\pi}{2} \right) + i \operatorname{sen} \left(\frac{\pi}{2} \right) \right)$$

Exemplo 4 – Solução

$$z_1 = 6 \left(\cos \left(\frac{2\pi}{3} \right) + i \operatorname{sen} \left(\frac{2\pi}{3} \right) \right) \text{ e } z_2 = 3 \left(\cos \left(\frac{\pi}{2} \right) + i \operatorname{sen} \left(\frac{\pi}{2} \right) \right)$$

$$\rho_1 = 6 \quad \varphi_1 = \frac{2\pi}{3} \quad \rho_2 = 3 \quad \varphi_2 = \frac{\pi}{2}$$

$$\rho = \frac{\rho_1}{\rho_2} = \frac{6}{3} = 2$$

$$\varphi = \varphi_1 - \varphi_2 = \frac{2\pi}{3} - \frac{\pi}{2} = \frac{4\pi - 3\pi}{6} = \frac{\pi}{6}$$

$$\begin{aligned} z = \frac{z_1}{z_2} &= \rho (\cos(\varphi) + i \operatorname{sen}(\varphi)) = 2 \left(\cos \left(\frac{\pi}{6} \right) + i \operatorname{sen} \left(\frac{\pi}{6} \right) \right) \\ &= 2 \left(\frac{\sqrt{3}}{2} + \frac{i}{2} \right) = 3 + i \end{aligned}$$

Conteúdo

Forma Polar ou Trigonométrica de um Número Complexo

Multiplicação

Divisão

Lista Mínima

Lista Mínima

Atenção: A prova é baseada no livro, não nas apresentações